Overview

1. Recap
2. Why rank?
3. More on cosine
4. The complete search system
5. Implementation of ranking
Outline

1. Recap
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Term frequency weight

- The log frequency weight of term $t$ in $d$ is defined as follows

$$w_{t,d} = \begin{cases} 
1 + \log_{10} tf_{t,d} & \text{if } tf_{t,d} > 0 \\
0 & \text{otherwise}
\end{cases}$$
The document frequency $df_t$ is defined as the number of documents that $t$ occurs in.

We define the \textit{idf weight} of term $t$ as follows:

$$\text{idf}_t = \log_{10} \frac{N}{df_t}$$

\text{idf} is a measure of the \textit{informativeness} of the term.
The tf-idf weight of a term is the product of its tf weight and its idf weight.

\[ w_{t,d} = (1 + \log \text{tf}_{t,d}) \cdot \log \frac{N}{\text{df}_t} \]
Cosine similarity between query and document

$$\cos(\vec{q}, \vec{d}) = \text{SIM}(\vec{q}, \vec{d}) = \frac{\vec{q}}{|\vec{q}|} \cdot \frac{\vec{d}}{|\vec{d}|} = \sum_{i=1}^{|V|} \frac{q_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2}} \cdot \frac{d_i}{\sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

- $q_i$ is the tf-idf weight of term $i$ in the query.
- $d_i$ is the tf-idf weight of term $i$ in the document.
- $|\vec{q}|$ and $|\vec{d}|$ are the lengths of $\vec{q}$ and $\vec{d}$.
- $\vec{q}/|\vec{q}|$ and $\vec{d}/|\vec{d}|$ are length-1 vectors (normalized).
Cosine similarity illustrated
### tf-idf example: lnc.ltn

Query: “best car insurance”. Document: “car insurance auto insurance”.

<table>
<thead>
<tr>
<th>word</th>
<th>tf-raw</th>
<th>tf-wght</th>
<th>df</th>
<th>idf</th>
<th>tf-idf weight</th>
<th>document</th>
<th>tf-raw</th>
<th>tf-wght</th>
<th>tf-wght</th>
<th>n’lized</th>
<th>product</th>
</tr>
</thead>
<tbody>
<tr>
<td>auto</td>
<td>0</td>
<td>0</td>
<td>5000</td>
<td>2.3</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.52</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>best</td>
<td>1</td>
<td>1</td>
<td>50000</td>
<td>1.3</td>
<td>1.3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>car</td>
<td>1</td>
<td>1</td>
<td>10000</td>
<td>2.0</td>
<td>2.0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.52</td>
<td>1.04</td>
<td>1.04</td>
</tr>
<tr>
<td>insurance</td>
<td>1</td>
<td>1</td>
<td>1000</td>
<td>3.0</td>
<td>3.0</td>
<td>2</td>
<td>1.3</td>
<td>1.3</td>
<td>0.68</td>
<td>2.04</td>
<td>2.04</td>
</tr>
</tbody>
</table>

Key to columns: tf-raw: raw (unweighted) term frequency, tf-wght: logarithmically weighted term frequency, df: document frequency, idf: inverse document frequency, weight: the final weight of the term in the query or document, n’lized: document weights after cosine normalization, product: the product of final query weight and final document weight.

\[
\sqrt{1^2 + 0^2 + 1^2 + 1.3^2} \approx 1.92 \\
\frac{1}{1.92} \approx 0.52 \\
\frac{1.3}{1.92} \approx 0.68 \quad \text{Final similarity score between query and document: } \sum_i w_{qi} \cdot w_{di} = 0 + 0 + 1.04 + 2.04 = 3.08
\]
Take-away today

- The importance of ranking: User studies at Google
- Length normalization: Pivot normalization
- The complete search system
- Implementation of ranking
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Why is ranking so important?

- Last lecture: Problems with unranked retrieval
  - Users want to look at a few results – not thousands.
  - It’s very hard to write queries that produce a few results.
  - Even for expert searchers
  - → Ranking is important because it effectively reduces a large set of results to a very small one.

- Next: More data on “users only look at a few results”
Empirical investigation of the effect of ranking

- The following slides are from Dan Russell’s JCDL 2007 talk
- Dan Russell was the “Über Tech Lead for Search Quality & User Happiness” at Google.
- How can we measure how important ranking is?
- Observe what searchers do when they are searching in a controlled setting
  - Videotape them
  - Ask them to “think aloud”
  - Interview them
  - Eye-track them
  - Time them
  - Record and count their clicks
So.. Did you notice the FTD official site?

To be honest, I didn’t even look at that.
At first I saw “from $20” and $20 is what I was looking for.
To be honest, 1800-flowers is what I’m familiar with and why I went there next even though I kind of assumed they wouldn’t have $20 flowers.

And you knew they were expensive?

I knew they were expensive but I thought “hey, maybe they’ve got some flowers for under $20 here…”

But you didn’t notice the FTD?

No I didn’t, actually… that’s really funny.
Rapidly scanning the results

Note scan pattern:

Page 3:
Result 1
Result 2
Result 3
Result 4
Result 3
Result 2
Result 4
Result 5
Result 6 <click>

Q: Why do this?
A: What’s learned later influences judgment of earlier content.
Kinds of behaviors we see in the data

- **Short / Nav**
- **Topic exploration**
- **Topic switch**
  - New topic
- **Methodical results exploration**
- **Query reform**

**Multitasking**

**Task 2**

**Stacking behavior**
How many links do users view?

Total number of abstracts viewed per page

Mean: 3.07    Median/Mode: 2.00

Dip after page break
Looking vs. Clicking

- Users view results one and two more often / thoroughly
- Users click most frequently on result one
Order of presentation influences where users look \textbf{AND} where they click.
Importance of ranking: Summary

- **Viewing abstracts:** Users are a lot more likely to read the abstracts of the top-ranked pages (1, 2, 3, 4) than the abstracts of the lower ranked pages (7, 8, 9, 10).

- **Clicking:** Distribution is even more skewed for clicking.
  - In 1 out of 2 cases, users click on the top-ranked page.
  - Even if the top-ranked page is not relevant, 30% of users will click on it.

→ Getting the ranking right is very important.

→ Getting the top-ranked page right is most important.
Exercise

- Ranking is also one of the high barriers to entry for competitors to established players in the search engine market.
- Why?
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Why distance is a bad idea

The Euclidean distance of $\vec{q}$ and $\vec{d}_2$ is large although the distribution of terms in the query $q$ and the distribution of terms in the document $d_2$ are very similar. That’s why we do length normalization or, equivalently, use cosine to compute query-document matching scores.
Exercise: A problem for cosine normalization

- Query $q$: “anti-doping rules Beijing 2008 Olympics”
- Compare three documents
  - $d_1$: a short document on anti-doping rules at 2008 Olympics
  - $d_2$: a long document that consists of a copy of $d_1$ and 5 other news stories, all on topics different from Olympics/anti-doping
  - $d_3$: a short document on anti-doping rules at the 2004 Athens Olympics

- What ranking do we expect in the vector space model?
- What can we do about this?
Pivot normalization

- Cosine normalization produces weights that are too large for short documents and too small for long documents (on average).
- Adjust cosine normalization by linear adjustment: “turning” the average normalization on the pivot
- Effect: Similarities of short documents with query decrease; similarities of long documents with query increase.
- This removes the unfair advantage that short documents have.
Predicted and true probability of relevance

Relevance vs Retrieval with cosine normalization

"true" relevance

crossing point

cosine norm

"probability" of relevance/retrieval

document length

source: Lillian Lee
Pivot normalization

Cosine Normalization

Pivoted Normalization

\[ \alpha \]

slope = \tan(\alpha)

source: Lillian Lee
### Pivoted normalization: Amit Singhal’s experiments

<table>
<thead>
<tr>
<th>Cosine</th>
<th>Pivoted Cosine Normalization</th>
<th>Slope</th>
<th>0.60</th>
<th>0.65</th>
<th>0.70</th>
<th>0.75</th>
<th>0.80</th>
</tr>
</thead>
<tbody>
<tr>
<td>6,526</td>
<td>6,342</td>
<td>6,458</td>
<td>6,574</td>
<td></td>
<td>6,629</td>
<td></td>
<td>6,671</td>
</tr>
<tr>
<td>0.2840</td>
<td>0.3024</td>
<td>0.3097</td>
<td>0.3144</td>
<td></td>
<td>0.3171</td>
<td></td>
<td>0.3162</td>
</tr>
<tr>
<td>Improvement</td>
<td>+ 6.5%</td>
<td>+ 9.0%</td>
<td>+10.7%</td>
<td></td>
<td>+11.7%</td>
<td></td>
<td>+11.3%</td>
</tr>
</tbody>
</table>

(relevant documents retrieved and (change in) average precision)
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Complete search system
Tiered indexes

- Basic idea:
  - Create several tiers of indexes, corresponding to importance of indexing terms
  - During query processing, start with highest-tier index
  - If highest-tier index returns at least \( k \) (e.g., \( k = 100 \)) results: stop and return results to user
  - If we’ve only found \(< k\) hits: repeat for next index in tier cascade

- Example: two-tier system
  - Tier 1: Index of all titles
  - Tier 2: Index of the rest of documents
  - Pages containing the search words in the title are better hits than pages containing the search words in the body of the text.
Tiered index

Tier 1
- auto
- best
- car
- insurance

Tier 2
- auto
- best
- car
- insurance

Tier 3
- auto
- best
- car
- insurance
The use of tiered indexes is believed to be one of the reasons that Google search quality was significantly higher initially (2000/01) than that of competitors.

(Along with PageRank, use of anchor text and proximity constraints)
Complete search system
Components we have introduced thus far

- Document preprocessing (linguistic and otherwise)
- Positional indexes
- Tiered indexes
- Spelling correction
- k-gram indexes for wildcard queries and spelling correction
- Query processing
- Document scoring
Components we haven’t covered yet

- Document cache: we need this for generating snippets (dynamic summaries)
- Zone indexes: They separate the indexes for different zones: the body of the document, all highlighted text in the document, anchor text, text in metadata fields etc
- Machine-learned ranking functions
- Proximity ranking (e.g., rank documents in which the query terms occur in the same local window higher than documents in which the query terms occur far from each other)
- Query parser
Vector space retrieval: Interactions

- How do we combine phrase retrieval with vector space retrieval?
- We do not want to compute document frequency / idf for every possible phrase. Why?
- How do we combine Boolean retrieval with vector space retrieval?
  - For example: “+”-constraints and “-”-constraints
  - Postfiltering is simple, but can be very inefficient – no easy answer.
- How do we combine wild cards with vector space retrieval?
- Again, no easy answer
Exercise

- Design criteria for tiered system
  - Each tier should be an order of magnitude smaller than the next tier.
  - The top 100 hits for most queries should be in tier 1, the top 100 hits for most of the remaining queries in tier 2 etc.
  - We need a simple test for “can I stop at this tier or do I have to go to the next one?”
    - There is no advantage to tiering if we have to hit most tiers for most queries anyway.

- Consider a two-tier system where the first tier indexes titles and the second tier everything.

- Question: Can you think of a better way of setting up a multitier system? Which “zones” of a document should be indexed in the different tiers (title, body of document, others?)? What criterion do you want to use for including a document in tier 1?
Now we also need term frequencies in the index

Brutus  →  1 , 2  7 , 3  83 , 1  87 , 2  ...  

Caesar  →  1 , 1  5 , 1  13 , 1  17 , 1  ...  

Calpurnia  →  7 , 1  8 , 2  40 , 1  97 , 3  

We also need positions. Not shown here.
Term frequencies in the inverted index

- Thus: In each posting, store $tf_{t,d}$ in addition to docID $d$.
- As an integer frequency, not as a (log-)weighted real number
  ... 
- ... because real numbers are difficult to compress.
- Overall, additional space requirements are small: a byte per posting or less
How do we compute the top $k$ in ranking?

We usually don’t need a complete ranking.
We just need the top $k$ for a small $k$ (e.g., $k = 100$).
If we don’t need a complete ranking, is there an efficient way of computing just the top $k$?

Naive:
- Compute scores for all $N$ documents
- Sort
- Return the top $k$

Not very efficient

Alternative: min heap
Use min heap for selecting top $k$ out of $N$

- A binary min heap is a binary tree in which each node’s value is less than the values of its children.
- Takes $O(N \log k)$ operations to construct (where $N$ is the number of documents) . . .
- . . . then read off $k$ winners in $O(k \log k)$ steps
Binary min heap

0.6

0.85

0.9

0.97

0.7

0.8

0.95
Selecting top $k$ scoring documents in $O(N \log k)$

- **Goal:** Keep the top $k$ documents seen so far
- **Use a binary min heap**
- **To process a new document $d'$ with score $s'$:**
  - Get current minimum $h_m$ of heap ($O(1)$)
  - If $s' \leq h_m$ skip to next document
  - If $s' > h_m$ heap-delete-root ($O(\log k)$)
  - Heap-add $d'/s'$ ($O(\log k)$)
Even more efficient computation of top $k$?

- Ranking has time complexity $O(N)$ where $N$ is the number of documents.
- Optimizations reduce the constant factor, but they are still $O(N)$, $N > 10^{10}$
- Are there sublinear algorithms?
- What we’re doing in effect: solving the $k$-nearest neighbor (kNN) problem for the query vector (= query point).
- There are no general solutions to this problem that are sublinear.
More efficient computation of top $k$: Heuristics

- **Idea 1: Reorder postings lists**
  - Instead of ordering according to docID ... 
  - ... order according to some measure of “expected relevance”.

- **Idea 2: Heuristics to prune the search space**
  - Not guaranteed to be correct ... 
  - ... but fails rarely.
  - In practice, close to constant time.
  - For this, we’ll need the concepts of document-at-a-time processing and term-at-a-time processing.
So far: postings lists have been ordered according to docID.

Alternative: a query-independent measure of “goodness” of a page

Example: PageRank $g(d)$ of page $d$, a measure of how many “good” pages hyperlink to $d$ (chapter 21)

Order documents in postings lists according to PageRank: $g(d_1) > g(d_2) > g(d_3) > \ldots$

Define composite score of a document:

$$\text{net-score}(q, d) = g(d) + \cos(q, d)$$

This scheme supports early termination: We do not have to process postings lists in their entirety to find top $k$. 

Order documents in postings lists according to PageRank:
\[ g(d_1) > g(d_2) > g(d_3) > \ldots \]

Define composite score of a document:
\[ \text{net-score}(q, d) = g(d) + \cos(q, d) \]

Suppose: (i) \( g \rightarrow [0, 1] \); (ii) \( g(d) < 0.1 \) for the document \( d \) we’re currently processing; (iii) smallest top \( k \) score we’ve found so far is 1.2

Then all subsequent scores will be \( < 1.1 \).

So we’ve already found the top \( k \) and can stop processing the remainder of postings lists.

Questions?
Document-at-a-time processing

- Both docID-ordering and PageRank-ordering impose a consistent ordering on documents in postings lists.
- Computing cosines in this scheme is document-at-a-time.
- We complete computation of the query-document similarity score of document $d_i$ before starting to compute the query-document similarity score of $d_{i+1}$.
- Alternative: term-at-a-time processing
Weight-sorted postings lists

- Idea: don’t process postings that contribute little to final score
- Order documents in postings list according to weight
- Simplest case: normalized tf-idf weight (rarely done: hard to compress)
- Documents in the top $k$ are likely to occur early in these ordered lists.
- Early termination while processing postings lists is unlikely to change the top $k$.
- But:
  - We no longer have a consistent ordering of documents in postings lists.
  - We no longer can employ document-at-a-time processing.
Term-at-a-time processing

- Simplest case: completely process the postings list of the first query term
- Create an accumulator for each docID you encounter
- Then completely process the postings list of the second query term
- ... and so forth
Term-at-a-time processing

**CosineScore**($q$)

1. `float Scores[N] = 0`
2. `float Length[N]`
3. **for each** query term $t$
4. **do** calculate $w_{t,q}$ and fetch postings list for $t$
5. **for each** pair($d$, $tf_{t,d}$) in postings list
6. **do** $Scores[d] + = w_{t,d} \times w_{t,q}$
7. Read the array $Length$
8. **for each** $d$
9. **do** $Scores[d] = Scores[d] / Length[d]$
10. **return** Top $k$ components of $Scores[]$

The elements of the array “Scores” are called **accumulators**.
For the web (20 billion documents), an array of accumulators $A$ in memory is infeasible.

Thus: Only create accumulators for docs occurring in postings lists

This is equivalent to: Do not create accumulators for docs with zero scores (i.e., docs that do not contain any of the query terms)
Accumulators: Example

\[
\begin{array}{c|c|c|c|c|c}
\text{Brutus} & 1,2 & 7,3 & 83,1 & 87,2 & \ldots \\
\text{Caesar} & 1,1 & 5,1 & 13,1 & 17,1 & \ldots \\
\text{Calpurnia} & 7,1 & 8,2 & 40,1 & 97,3
\end{array}
\]

- For query: [Brutus Caesar]:
  - Only need accumulators for 1, 5, 7, 13, 17, 83, 87
  - Don’t need accumulators for 3, 8 etc.
We can enforce conjunctive search (a la Google): only consider documents (and create accumulators) if all terms occur.

Example: just one accumulator for [Brutus Caesar] in the example above . . .

. . . because only $d_1$ contains both words.
Implementation of ranking: Summary

- Ranking is very expensive in applications where we have to compute similarity scores for all documents in the collection.
- In most applications, the vast majority of documents have similarity score 0 for a given query → lots of potential for speeding things up.
- However, there is no fast nearest neighbor algorithm that is guaranteed to be correct even in this scenario.
- In practice: use heuristics to prune search space – usually works very well.
Take-away today

- The importance of ranking: User studies at Google
- Length normalization: Pivot normalization
- The complete search system
- Implementation of ranking
Resources

- Chapters 6 and 7 of IIR
- Resources at http://cislmu.org
  - How Google tweaks its ranking function
  - Interview with Google search guru Udi Manber
  - Amit Singhal on Google ranking
  - SEO perspective: ranking factors
  - Yahoo Search BOSS: Opens up the search engine to developers. For example, you can rerank search results.
  - Compare Google and Yahoo ranking for a query
  - How Google uses eye tracking for improving search