Introduction to Information Retrieval
http://informationretrieval.org

IIR 7: Scores in a Complete Search System

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Overview

1. Recap
2. Why rank?
3. More on cosine
4. The complete search system
5. Implementation of ranking
Outline

1. Recap

2. Why rank?

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Term frequency weight

The log frequency weight of term $t$ in $d$ is defined as follows:

$$w_{t,d} = \begin{cases} 
1 + \log_{10} tf_{t,d} & \text{if } tf_{t,d} > 0 \\
0 & \text{otherwise}
\end{cases}$$
The document frequency $df_t$ is defined as the number of documents that $t$ occurs in.

We define the idf weight of term $t$ as follows:

$$idf_t = \log_{10} \frac{N}{df_t}$$

idf is a measure of the informativeness of the term.
The tf-idf weight of a term is the \textit{product of its tf weight and its idf weight}.

\[ w_{t,d} = (1 + \log \text{tf}_{t,d}) \cdot \log \frac{N}{\text{df}_t} \]
Cosine similarity between query and document

\[
\cos(\vec{q}, \vec{d}) = \frac{\vec{q} \cdot \vec{d}}{|\vec{q}| \cdot |\vec{d}|} = \frac{\sum_{i=1}^{n} q_i}{\sqrt{\sum_{i=1}^{n} q_i^2}} \cdot \frac{\sum_{i=1}^{n} d_i}{\sqrt{\sum_{i=1}^{n} d_i^2}}
\]

- \(q_i\) is the tf-idf weight of term \(i\) in the query.
- \(d_i\) is the tf-idf weight of term \(i\) in the document.
- \(|\vec{q}|\) and \(|\vec{d}|\) are the lengths of \(\vec{q}\) and \(\vec{d}\).
- \(\vec{q}/|\vec{q}|\) and \(\vec{d}/|\vec{d}|\) are length-1 vectors (= normalized).
Cosine similarity illustrated

\[ \vec{v}(d_1) \]

\[ \vec{v}(d_2) \]

\[ \vec{v}(d_3) \]

\[ \vec{v}(q) \]

\[ \theta \]

POOR

RICH
### tf-idf example: inc.ltn

Query: “best car insurance”. Document: “car insurance auto insurance”.

<table>
<thead>
<tr>
<th>word</th>
<th>query</th>
<th>document</th>
<th>product</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>tf-raw</td>
<td>tf-wght</td>
<td>df</td>
</tr>
<tr>
<td>auto</td>
<td>0</td>
<td>0</td>
<td>5000</td>
</tr>
<tr>
<td>best</td>
<td>1</td>
<td>1</td>
<td>50000</td>
</tr>
<tr>
<td>car</td>
<td>1</td>
<td>1</td>
<td>10000</td>
</tr>
<tr>
<td>insurance</td>
<td>1</td>
<td>1</td>
<td>1000</td>
</tr>
</tbody>
</table>

Key to columns: tf-raw: raw (unweighted) term frequency, tf-wght: logarithmically weighted term frequency, df: document frequency, idf: inverse document frequency, weight: the final weight of the term in the query or document, n’lized: document weights after cosine normalization, product: the product of final query weight and final document weight.

\[ \sqrt{1^2 + 0^2 + 1^2 + 1.3^2} \approx 1.92 \]
\[ 1/1.92 \approx 0.52 \]
\[ 1.3/1.92 \approx 0.68 \]

Final similarity score between query and document: \( \sum_i w_{qi} \cdot w_{di} = 0 + 0 + 1.04 + 2.04 = 3.08 \)
Take-away today
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- The importance of ranking: User studies at Google
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- The importance of ranking: User studies at Google
- Length normalization: Pivot normalization
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  → Ranking is important because it effectively reduces a large set of results to a very small one.
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- Next: More data on “users only look at a few results”
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  - Interview them
  - Eye-track them
  - Time them
  - Record and count their clicks
So.. Did you notice the FTD official site?

To be honest, I didn’t even look at that.
At first I saw “from $20” and $20 is what I was looking for.
To be honest, 1800-flowers is what I’m familiar with and why I went there next even though I kind of assumed they wouldn’t have $20 flowers

And you knew they were expensive?

I knew they were expensive but I thought “hey, maybe they’ve got some flowers for under $20 here…”

But you didn’t notice the FTD?

No I didn’t, actually… that’s really funny.
Rapidly scanning the results

Note scan pattern:

Page 3:  
Result 1  
Result 2  
Result 3  
Result 4  
Result 3  
Result 2  
Result 4  
Result 5  
Result 6 <click>

Q: Why do this?  
A: What’s learned later influences judgment of earlier content.
Kinds of behaviors we see in the data

- Short / Nav
- Topic exploration
- Topic switch
  - New topic
- Methodical results exploration
- Query reform
- Multitasking
  - Task 2
- Stacking behavior
How many links do users view?

Total number of abstracts viewed per page

Mean: 3.07  Median/Mode: 2.00
Looking vs. Clicking

- Users view results one and two more often / thoroughly
- Users click most frequently on result one
Order of presentation influences where users look AND where they click.
Importance of ranking: Summary
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→ **Getting the top-ranked page right is most important.**
Exercise

- Ranking is also one of the high barriers to entry for competitors to established players in the search engine market.

- Why?
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Why distance is a bad idea

The Euclidean distance of $\vec{q}$ and $\vec{d}_2$ is large although the distribution of terms in the query $q$ and the distribution of terms in the document $d_2$ are very similar.

That’s why we do length normalization or, equivalently, use cosine to compute query-document matching scores.
Exercise: A problem for cosine normalization

- Query \( q \): “anti-doping rules Beijing 2008 olympics”
- Compare three documents
  - \( d_1 \): a short document on anti-doping rules at 2008 Olympics
  - \( d_2 \): a long document that consists of a copy of \( d_1 \) and 5 other news stories, all on topics different from Olympics/anti-doping
  - \( d_3 \): a short document on anti-doping rules at the 2004 Athens Olympics
- What ranking do we expect in the vector space model?
- What can we do about this?
Pivot normalization

- Cosine normalization produces weights that are too large for short documents and too small for long documents (on average).
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- Effect: Similarities of short documents with query decrease; similarities of long documents with query increase.
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- Adjust cosine normalization by linear adjustment: “turning” the average normalization on the pivot

- Effect: Similarities of short documents with query decrease; similarities of long documents with query increase.

- This removes the unfair advantage that short documents have.
Predicted and true probability of relevance
Predicted and true probability of relevance

Source: Lillian Lee

Relevance vs Retrieval with cosine normalization

"true" relevance

crossing point

cosine norm

"probability" of relevance/retrieval
Pivot normalization
Pivot normalization

\[ \alpha \text{ slope} = \tan(\alpha) \]

source: Lillian Lee
Pivoted normalization: Amit Singhal’s experiments
### Pivoted normalization: Amit Singhal’s experiments

<table>
<thead>
<tr>
<th>Cosine</th>
<th>Pivoted Cosine Normalization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Slope</td>
</tr>
<tr>
<td></td>
<td>0.60</td>
</tr>
<tr>
<td>6,526</td>
<td>6,342</td>
</tr>
<tr>
<td>0.2840</td>
<td>0.3024</td>
</tr>
<tr>
<td>Improvement</td>
<td>+ 6.5%</td>
</tr>
</tbody>
</table>

(relevant documents retrieved and (change in) average precision)
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Complete search system
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- **Example: two-tier system**
  - **Tier 1:** Index of all titles
  - **Tier 2:** Index of the rest of documents
  - Pages containing the search words in the title are better hits than pages containing the search words in the body of the text.
Tiered index
Tiered index

Tier 1
- auto → Doc2
- best
- car → Doc1 → Doc3
- insurance → Doc2 → Doc3

Tier 2
- auto
- best → Doc1 → Doc3
- car
- insurance

Tier 3
- auto → Doc1
- best
- car → Doc2
- insurance
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- (along with PageRank, use of anchor text and proximity constraints)
Complete search system
Components we have introduced thus far

- Document preprocessing (linguistic and otherwise)
- Positional indexes
- Tiered indexes
- Spelling correction
- k-gram indexes for wildcard queries and spelling correction
- Query processing
- Document scoring
Components we haven’t covered yet

- Document cache: we need this for generating snippets (= dynamic summaries)
- Zone indexes: They separate the indexes for different zones: the body of the document, all highlighted text in the document, anchor text, text in metadata fields etc
- Machine-learned ranking functions
- Proximity ranking (e.g., rank documents in which the query terms occur in the same local window higher than documents in which the query terms occur far from each other)
- Query parser
Vector space retrieval: Interactions
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- How do we combine phrase retrieval with vector space retrieval?
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- Postfiltering is simple, but can be very inefficient – no easy answer.
- How do we combine wild cards with vector space retrieval?
- Again, no easy answer
Exercise

- Design criteria for tiered system
  - Each tier should be an order of magnitude smaller than the next tier.
  - The top 100 hits for most queries should be in tier 1, the top 100 hits for most of the remaining queries in tier 2 etc.
  - We need a simple test for “can I stop at this tier or do I have to go to the next one?”
    - There is no advantage to tiering if we have to hit most tiers for most queries anyway.

- Consider a two-tier system where the first tier indexes titles and the second tier everything.

- Question: Can you think of a better way of setting up a multitier system? Which “zones” of a document should be indexed in the different tiers (title, body of document, others?)? What criterion do you want to use for including a document in tier 1?
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Now we also need term frequencies in the index
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<table>
<thead>
<tr>
<th>Name</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brutus</td>
<td>1,2 7,3 83,1 87,2...</td>
</tr>
<tr>
<td>Caesar</td>
<td>1,1 5,1 13,1 17,1...</td>
</tr>
<tr>
<td>Calpurnia</td>
<td>7,1 8,2 40,1 97,3</td>
</tr>
</tbody>
</table>
Now we also need term frequencies in the index

| Term      | Index 1,2 | Index 7,3 | Index 83,1 | Index 87,2 |...
<table>
<thead>
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<th></th>
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term frequencies

We also need positions. Not shown here.
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  ...
- ...because real numbers are difficult to compress.
- Overall, additional space requirements are small: a byte per posting or less
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Naive:
- Compute scores for all $N$ documents
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- Alternative: min heap
Use min heap for selecting top $k$ out of $N$
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- Takes $O(N \log k)$ operations to construct (where $N$ is the number of documents) . . .
- . . . then read off $k$ winners in $O(k \log k)$ steps
Binary min heap

0.6

0.85
0.9
0.97

0.7
0.8
0.95

Schütze: Scores in a complete search system
Selecting top $k$ scoring documents in $O(N \log k)$

- **Goal:** Keep the top $k$ documents seen so far
- **Use a binary min heap**
- **To process a new document $d'$ with score $s'$:**
  - Get current minimum $h_m$ of heap ($O(1)$)
  - If $s' \leq h_m$ skip to next document
  - If $s' > h_m$ heap-delete-root ($O(\log k)$)
  - Heap-add $d'/s'$ ($O(\log k)$)
Even more efficient computation of top $k$?

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- Are there sublinear algorithms?
- What we’re doing in effect: solving the $k$-nearest neighbor (kNN) problem for the query vector (= query point).
- There are no general solutions to this problem that are sublinear.
More efficient computation of top $k$: Heuristics
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- Idea 1: Reorder postings lists
More efficient computation of top $k$: Heuristics

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  - In practice, close to constant time.
  - For this, we’ll need the concepts of document-at-a-time processing and term-at-a-time processing.
Non-docID ordering of postings lists

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- This scheme supports early termination: We do not have to process postings lists in their entirety to find top $k$. 
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Questions?
Document-at-a-time processing
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- Alternative: term-at-a-time processing
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- But:
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- → Early termination while processing postings lists is unlikely to change the top $k$.
- But:
  - We no longer have a consistent ordering of documents in postings lists.
  - We no longer can employ document-at-a-time processing.
Term-at-a-time processing
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- Simplest case: completely process the postings list of the first query term
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- Create an accumulator for each docID you encounter
- Then completely process the postings list of the second query term
Term-at-a-time processing

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- Create an accumulator for each docID you encounter
- Then completely process the postings list of the second query term
- ... and so forth
| Recap | Why rank? | More on cosine | The complete search system | Implementation of ranking |

**Term-at-a-time processing**
term-at-a-time processing

\textbf{CosineScore}(q)

1. \[ \text{float } \text{Scores}[N] = 0 \]
2. \[ \text{float } \text{Length}[N] \]
3. \textbf{for each query term } t
4. \textbf{do calculate } w_{t,q} \text{ and fetch postings list for } t
5. \hspace{1em} \textbf{for each } \text{pair}(d, tf_{t,d}) \text{ in postings list}
6. \hspace{2em} \textbf{do } \text{Scores}[d] + = w_{t,d} \times w_{t,q}
7. \text{Read the array } \text{Length}
8. \textbf{for each } d
9. \hspace{1em} \textbf{do } \text{Scores}[d] = \text{Scores}[d] / \text{Length}[d]
10. \textbf{return Top } k \text{ components of } \text{Scores}[]

The elements of the array “Scores” are called \textit{accumulators}. 
Accumulators

- For the web (20 billion documents), an array of accumulators $A$ in memory is infeasible.
- Thus: Only create accumulators for docs occurring in postings lists
- This is equivalent to: Do not create accumulators for docs with zero scores (i.e., docs that do not contain any of the query terms)
Accumulators: Example

For query: [Brutus Caesar]:
- Only need accumulators for 1, 5, 7, 13, 17, 83, 87
- Don’t need accumulators for 3, 8 etc.
Enforcing conjunctive search
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- . . . because only $d_1$ contains both words.
Implementation of ranking: Summary
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Implementation of ranking: Summary

- Ranking is **very expensive** in applications where we have to compute similarity scores for all documents in the collection.
- In most applications, the vast majority of documents have *similarity score 0* for a given query → lots of potential for speeding things up.
- However, there is **no fast nearest neighbor algorithm** that is guaranteed to be correct even in this scenario.
- In practice: use **heuristics** to prune search space – usually works very well.
Take-away today

- The importance of ranking: User studies at Google
- Length normalization: Pivot normalization
- The complete search system
- Implementation of ranking
Resources

- Chapters 6 and 7 of IIR
- Resources at http://cislmu.org
  - How Google tweaks its ranking function
  - Interview with Google search guru Udi Manber
  - Amit Singhal on Google ranking
  - SEO perspective: ranking factors
  - Yahoo Search BOSS: Opens up the search engine to developers. For example, you can rerank search results.
  - Compare Google and Yahoo ranking for a query
  - How Google uses eye tracking for improving search