

# Introduction to Information Retrieval

<http://informationretrieval.org>

## IIR 16: Flat Clustering

Hinrich Schütze

Center for Information and Language Processing, University of Munich

2014-06-11

# Overview

- 1 Recap
- 2 Clustering: Introduction
- 3 Clustering in IR
- 4 *K*-means
- 5 Evaluation
- 6 How many clusters?

# Outline

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- 2 Clustering: Introduction
- 3 Clustering in IR
- 4  $K$ -means
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# Learning to rank for zone scoring

Given query  $q$  and document  $d$ , weighted zone scoring assigns to the pair  $(q, d)$  a score in the interval  $[0,1]$  by computing a **linear combination** of document zone scores, where each zone contributes a value.

- Consider a set of documents, which have  $l$  zones
- Let  $g_1, \dots, g_l \in [0, 1]$ , such that  $\sum_{i=1}^l g_i = 1$
- For  $1 \leq i \leq l$ , let  $s_i$  be the Boolean score denoting a match (or non-match) between  $q$  and the  $i^{th}$  zone
  - $s_i = 1$  if a query term occurs in zone  $i$ , 0 otherwise

Weighted zone scoring aka ranked Boolean retrieval

Rank documents according to  $\sum_{i=1}^l g_i s_i$

**Learning to rank approach:** learn the weights  $g_i$  from training data

## Training set for learning to rank

$\Phi_j$	$d_j$	$q_j$	$s_T$	$s_B$	$r(d_j, q_j)$
$\Phi_1$	37	linux	1	1	Relevant
$\Phi_2$	37	penguin	0	1	Nonrelevant
$\Phi_3$	238	system	0	1	Relevant
$\Phi_4$	238	penguin	0	0	Nonrelevant
$\Phi_5$	1741	kernel	1	1	Relevant
$\Phi_6$	2094	driver	0	1	Relevant
$\Phi_7$	3194	driver	1	0	Nonrelevant

# Summary of learning to rank approach

- The problem of making a binary relevant/nonrelevant judgment is cast as a classification or regression problem, based on a training set of query-document pairs and associated relevance judgments.
- In principle, any method learning a classifier (including least squares regression) can be used to find this line.
- Big advantage of learning to rank: we can avoid hand-tuning scoring functions and simply learn them from training data.
- Bottleneck of learning to rank: the cost of maintaining a representative set of training examples whose relevance assessments must be made by humans.

## LTR features used by Microsoft Research (1)

- Zones: body, anchor, title, url, whole document
- Features derived from standard IR models: query term number, query term ratio, length, idf, sum of term frequency, min of term frequency, max of term frequency, mean of term frequency, variance of term frequency, sum of length normalized term frequency, min of length normalized term frequency, max of length normalized term frequency, mean of length normalized term frequency, variance of length normalized term frequency, sum of tf-idf, min of tf-idf, max of tf-idf, mean of tf-idf, variance of tf-idf, boolean model, BM25

## LTR features used by Microsoft Research (2)

- Language model features: LMIR.ABS, LMIR.DIR, LMIR.JM
- Web-specific features: number of slashes in url, length of url, inlink number, outlink number, PageRank, SiteRank
- Spam features: QualityScore
- Usage-based features: query-url click count, url click count, url dwell time



# Ranking SVMs

- Vector of feature differences:  $\Phi(d_i, d_j, q) = \psi(d_i, q) - \psi(d_j, q)$
- By hypothesis, one of  $d_i$  and  $d_j$  has been judged more relevant.
- Notation: We write  $d_i \prec d_j$  for “ $d_i$  precedes  $d_j$  in the results ordering”.
- If  $d_i$  is judged more relevant than  $d_j$ , then we will assign the vector  $\Phi(d_i, d_j, q)$  the class  $y_{ijq} = +1$ ; otherwise  $-1$ .
- This gives us a training set of pairs of vectors and “precedence indicators”. Each of the vectors is computed as the difference of two document-query vectors.
- We can then train an SVM on this training set with the goal of obtaining a classifier that returns

$$\vec{w}^T \Phi(d_i, d_j, q) > 0 \quad \text{iff} \quad d_i \prec d_j$$

# Take-away today

- What is clustering?
- Applications of clustering in information retrieval
- *K*-means algorithm
- Evaluation of clustering
- How many clusters?



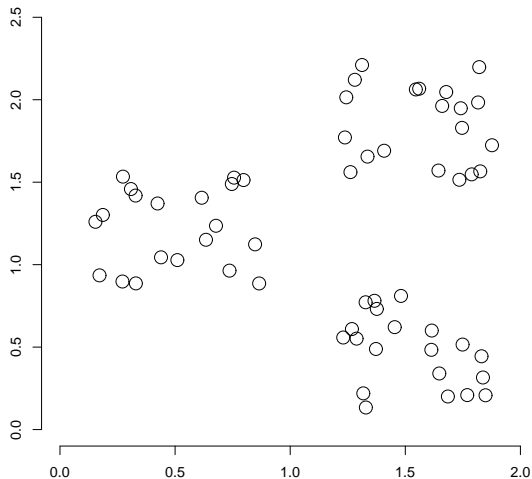
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# Clustering: Definition

- (Document) clustering is the process of **grouping a set of documents into clusters of similar documents**.
- Documents within a cluster should be similar.
- Documents from different clusters should be dissimilar.
- Clustering is the most common form of **unsupervised** learning.
- Unsupervised = there are no labeled or annotated data.

## Data set with clear cluster structure



Propose  
algorithm  
for finding  
the cluster  
structure in  
this  
example

# Classification vs. Clustering

- Classification: supervised learning
- Clustering: unsupervised learning
- Classification: Classes are **human-defined** and part of the input to the learning algorithm.
- Clustering: Clusters are **inferred from the data** without human input.
  - However, there are many ways of influencing the outcome of clustering: number of clusters, similarity measure, representation of documents, ...



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# The cluster hypothesis

**Cluster hypothesis.** Documents in the same cluster behave similarly with respect to relevance to information needs. All

applications of clustering in IR are based (directly or indirectly) on the cluster hypothesis. Van Rijsbergen's original wording (1979):

“closely associated documents tend to be relevant to the same requests” .





# Applications of clustering in IR

application	what is clustered?	benefit
search result clustering	search results	more effective information presentation to user
Scatter-Gather	(subsets of) collection	alternative user interface: "search without typing"
collection clustering	collection	effective information presentation for exploratory browsing
cluster-based retrieval	collection	higher efficiency: faster search

# Search result clustering for better navigation

**Vivísimo**    [Advanced Search](#) [Help](#)

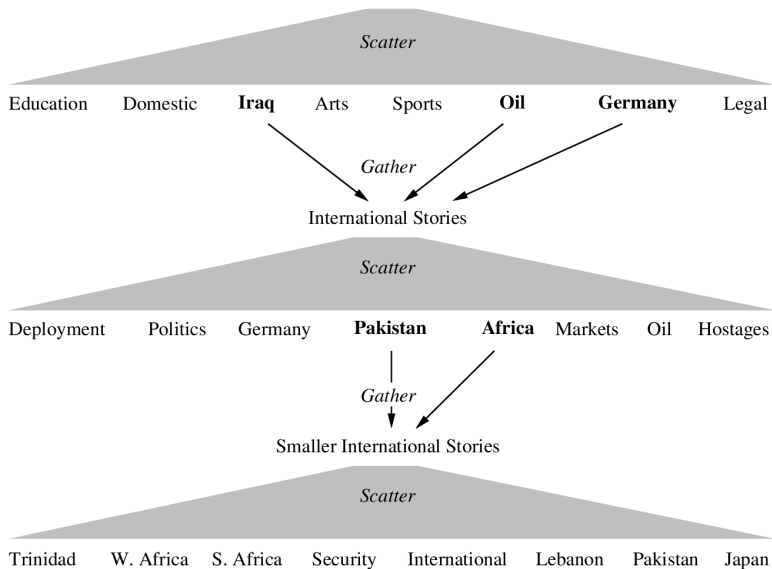
**Clustered Results** Top 208 results of at least 20,373,974 retrieved for the query **jaguar** (Details)

- Jaguar** (208)
  - Cars** (74)
  - Club** (34)
  - Cat** (23)
  - Animal** (13)
  - Restoration** (10)
  - Mac OS X** (8)
  - Jaguar Model** (8)
  - Request** (5)
  - Mark Webber** (6)
  - Maya** (5)
  - [More](#)

Find in clusters:

- [Jag-lovers - THE source for all Jaguar information](#) [new window] [iframe] [cache] [preview] [clusters]  
... Internet! Serving Enthusiasts since 1993 The Jag-lovers Web Currently with 40661 members The Premier **Jaguar** Cars web resource for all enthusiasts Lists and Forums Jag-lovers originally evolved around its ...  
[www.jag-lovers.org](http://www.jag-lovers.org) - Open Directory 2, Wisenut 6, Ask Jeeves 8, MSN 9, Looksmart 12, MSN Search 18
- [Jaguar Cars](#) [new window] [iframe] [cache] [preview] [clusters]  
[...] redirected to [www.jaguar.com](http://www.jaguar.com)  
[www.jaguarcars.com](http://www.jaguarcars.com) - Looksmart 1, MSN 2, Lycos 3, Wisenut 6, MSN Search 9, MSN 29
- <http://www.jaguar.com/> [new window] [iframe] [preview] [clusters]  
[www.jaguar.com](http://www.jaguar.com) - MSN 1, Ask Jeeves 1, MSN Search 3, Lycos 9
- [Apple - Mac OS X](#) [new window] [iframe] [preview] [clusters]  
Learn about the new OS X Server, designed for the Internet, digital media and workgroup management  
Download a technical factsheet.  
[www.apple.com/macosx](http://www.apple.com/macosx) - Wisenut 1, MSN 3, Looksmart 26


# Scatter-Gather



# Global navigation: Yahoo

**YAHOO! DIRECTORY** Search:  the Web |  the Directory |  this category  Search

**Society and Culture**  
Directory > Society and Culture

 **Culture**  
[www.Dealtime.com](http://www.Dealtime.com) Shop and save on Magazines. SPONSOR RE

CATEGORIES [\(What's This?\)](#)

---

**Most Popular Society and Culture**

- [Crime](#) (5453) NEWS!
- [Cultures and Groups](#) (11025) NEWS!
- [Environment and Nature](#) (8558) NEWS!
- [Families](#) (1215)
- [Food and Drink](#) (9776) NEWS!
- [Holidays and Observances](#) (3333)
- [Issues and Causes](#) (4842)
- [Mythology and Folklore](#) (984)
- [People](#) (16351)
- [Relationships](#) (595)
- [Religion and Spirituality](#) (37533)
- [Sexuality](#) (2812) NEWS!

**Additional Society and Culture Categories**

- [Advice](#) (48)
- [Chats and Forums](#) (27)
- [Cultural Policy](#) (10)
- [Death and Dying](#) (394)
- [Disabilities](#) (1293)
- [Employment and Work@](#)
- [Etiquette](#) (54)
- [Events](#) (27)
- [Fashion@](#)
- [Gender](#) (21)
- [Home and Garden](#) (1080) NEWS!
- [Magazines](#) (164)
- [Museums and Exhibits](#) (6052)
- [Pets@](#)
- [Reunions](#) (228)
- [Social Organizations](#) (338)
- [Web Directories](#) (6)
- [Weddings](#) (371)

SITE LISTINGS By Popularity | [Alphabetical](#) | [\(What's This?\)](#) Site

# Global navigation: MESH (upper level)

## MeSH Tree Structures - 2008

[Return to Entry Page](#)

1.  Anatomy [A]
2.  Organisms [B]
3.  Diseases [C]
  - [Bacterial Infections and Mycoses \[C01\] +](#)
  - [Virus Diseases \[C02\] +](#)
  - [Parasitic Diseases \[C03\] +](#)
  - [Neoplasms \[C04\] +](#)
  - [Musculoskeletal Diseases \[C05\] +](#)
  - [Digestive System Diseases \[C06\] +](#)
  - [Stomatognathic Diseases \[C07\] +](#)
  - [Respiratory Tract Diseases \[C08\] +](#)
  - [Otorhinolaryngologic Diseases \[C09\] +](#)
  - [Nervous System Diseases \[C10\] +](#)
  - [Eye Diseases \[C11\] +](#)
  - [Male Urogenital Diseases \[C12\] +](#)
  - [Female Urogenital Diseases and Pregnancy Complications \[C13\] +](#)
  - [Cardiovascular Diseases \[C14\] +](#)
  - [Hemic and Lymphatic Diseases \[C15\] +](#)
  - [Congenital, Hereditary, and Neonatal Diseases and Abnormalities \[C16\] +](#)
  - [Skin and Connective Tissue Diseases \[C17\] +](#)
  - [Nutritional and Metabolic Diseases \[C18\] +](#)
  - [Endocrine System Diseases \[C19\] +](#)
  - [Immune System Diseases \[C20\] +](#)
  - [Disorders of Environmental Origin \[C21\] +](#)
  - [Animal Diseases \[C22\] +](#)
  - [Pathological Conditions, Signs and Symptoms \[C23\] +](#)
4.  Chemicals and Drugs [D]
5.  Analytical, Diagnostic and Therapeutic Techniques and Equipment [E]
6.  Psychiatry and Psychology [F]
7.  Biological Sciences [G]
8.  Natural Sciences [H]
9.  Anthropology, Education, Sociology and Social Phenomena [I]
10.  Technology, Industry, Agriculture [J]
11.  Humanities [K]

# Global navigation: MESH (lower level)

## [Neoplasms \[C04\]](#)

[Cysts \[C04.182\] +](#)

[Hamartoma \[C04.445\] +](#)

### ► [Neoplasms by Histologic Type \[C04.557\]](#)

[Histiocytic Disorders, Malignant \[C04.557.227\] +](#)

[Leukemia \[C04.557.337\] +](#)

[Lymphatic Vessel Tumors \[C04.557.375\] +](#)

[Lymphoma \[C04.557.386\] +](#)

[Neoplasms, Complex and Mixed \[C04.557.435\] +](#)

[Neoplasms, Connective and Soft Tissue \[C04.557.450\] +](#)

[Neoplasms, Germ Cell and Embryonal \[C04.557.465\] +](#)

[Neoplasms, Glandular and Epithelial \[C04.557.470\] +](#)

[Neoplasms, Gonadal Tissue \[C04.557.475\] +](#)

[Neoplasms, Nerve Tissue \[C04.557.580\] +](#)

[Neoplasms, Plasma Cell \[C04.557.595\] +](#)

[Neoplasms, Vascular Tissue \[C04.557.645\] +](#)

[Nevi and Melanomas \[C04.557.665\] +](#)

[Odontogenic Tumors \[C04.557.695\] +](#)

[Neoplasms by Site \[C04.588\] +](#)

[Neoplasms, Experimental \[C04.619\] +](#)

[Neoplasms, Hormone-Dependent \[C04.626\]](#)

[Neoplasms, Multiple Primary \[C04.651\] +](#)

[Neoplasms, Post-Traumatic \[C04.666\]](#)

[Neoplasms, Radiation-Induced \[C04.682\] +](#)

[Neoplasms, Second Primary \[C04.692\]](#)

[Neoplastic Processes \[C04.697\] +](#)

[Neoplastic Syndromes, Hereditary \[C04.700\] +](#)

[Paraneoplastic Syndromes \[C04.730\] +](#)

[Precancerous Conditions \[C04.834\] +](#)

[Pregnancy Complications, Neoplastic \[C04.850\] +](#)

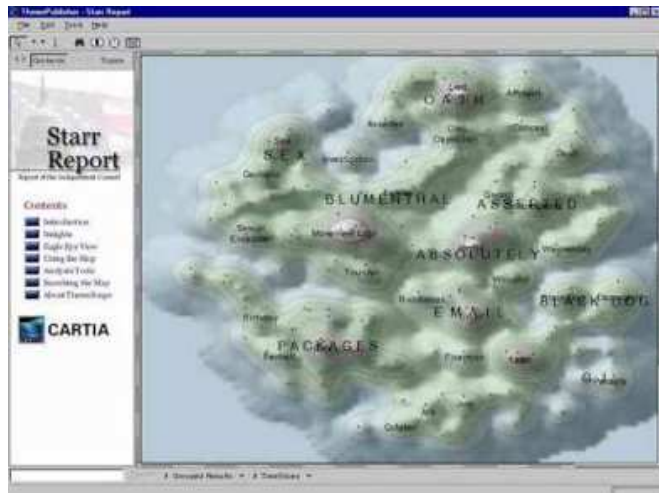
[Tumor Virus Infections \[C04.925\] +](#)

# Navigational hierarchies: Manual vs. automatic creation

- Note: Yahoo/MESH are **not** examples of clustering.
- But they are well known examples for using a global hierarchy for navigation.
- Some examples for global navigation/exploration based on clustering:
  - Cartia
  - Themescapes
  - Google News



# Global navigation combined with visualization (1)







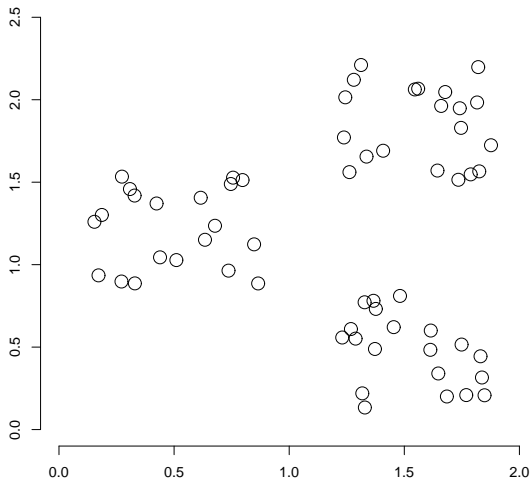
# Global clustering for navigation: Google News

<http://news.google.com>

# Clustering for improving recall

- To improve search recall:
  - Cluster docs in collection a priori
  - When a query matches a doc  $d$ , also return other docs in the cluster containing  $d$
- Hope: if we do this: the query “car” will also return docs containing “automobile”
  - Because the clustering algorithm groups together docs containing “car” with those containing “automobile” .
  - Both types of documents contain words like “parts”, “dealer”, “mercedes”, “road trip” . □

## Data set with clear cluster structure



Propose  
algorithm  
for finding  
the cluster  
structure in  
this  
example

# Desiderata for clustering

- General goal: put related docs in the same cluster, put unrelated docs in different clusters.
  - We'll see different ways of formalizing this.
- The number of clusters should be appropriate for the data set we are clustering.
  - Initially, we will assume the number of clusters  $K$  is given.
  - Later: Semiautomatic methods for determining  $K$
- Secondary goals in clustering
  - Avoid very small and very large clusters
  - Define clusters that are easy to explain to the user
  - Many others . . .



# Flat vs. Hierarchical clustering

- Flat algorithms
  - Usually start with a random (partial) partitioning of docs into groups
  - Refine iteratively
  - Main algorithm:  $K$ -means
- Hierarchical algorithms
  - Create a hierarchy
  - Bottom-up, agglomerative
  - Top-down, divisive



# Hard vs. Soft clustering

- Hard clustering: Each document belongs to **exactly one** cluster.
  - More common and easier to do
- Soft clustering: A document can belong to **more than one** cluster.
  - Makes more sense for applications like creating browsable hierarchies
  - You may want to put *sneakers* in two clusters:
    - sports apparel
    - shoes
  - You can only do that with a soft clustering approach.
- This class: flat, hard clustering
- Next time: hierarchical, hard clustering
- Next week: latent semantic indexing, a form of soft clustering



# Flat algorithms

- Flat algorithms compute a partition of  $N$  documents into a set of  $K$  clusters.
- Given: a set of documents and the number  $K$
- Find: a partition into  $K$  clusters that optimizes the chosen partitioning criterion
- Global optimization: exhaustively enumerate partitions, pick optimal one
  - Not tractable
- Effective heuristic method:  $K$ -means algorithm □



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# $K$ -means

- Perhaps the best known clustering algorithm
- Simple, works well in many cases
- Use as default / baseline for clustering documents



# Document representations in clustering

- Vector space model
- As in vector space classification, we measure relatedness between vectors by **Euclidean distance** ...
- ... which is almost equivalent to cosine similarity.
- Almost: centroids are not length-normalized.



# K-means: Basic idea

- Each cluster in K-means is defined by a **centroid**.
- Objective/partitioning criterion: **minimize the average squared difference from the centroid**
- Recall definition of centroid:

$$\vec{\mu}(\omega) = \frac{1}{|\omega|} \sum_{\vec{x} \in \omega} \vec{x}$$

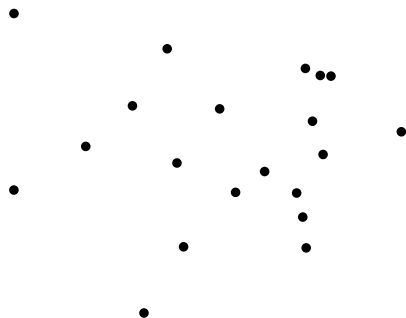
where we use  $\omega$  to denote a cluster.

- We try to find the minimum average squared difference by iterating two steps:
  - **reassignment**: assign each vector to its closest centroid
  - **recomputation**: recompute each centroid as the average of the vectors that were assigned to it in reassignment □

## K-means pseudocode ( $\mu_k$ is centroid of $\omega_k$ )

```
K-MEANS( $\{\vec{x}_1, \dots, \vec{x}_N\}, K$ )
1  ( $\vec{s}_1, \vec{s}_2, \dots, \vec{s}_K$ )  $\leftarrow$  SELECTRANDOMSEEDS( $\{\vec{x}_1, \dots, \vec{x}_N\}, K$ )
2  for  $k \leftarrow 1$  to  $K$ 
3  do  $\vec{\mu}_k \leftarrow \vec{s}_k$ 
4  while stopping criterion has not been met
5  do for  $k \leftarrow 1$  to  $K$ 
6    do  $\omega_k \leftarrow \{\}$ 
7    for  $n \leftarrow 1$  to  $N$ 
8    do  $j \leftarrow \arg \min_{j'} |\vec{\mu}_{j'} - \vec{x}_n|$ 
9       $\omega_j \leftarrow \omega_j \cup \{\vec{x}_n\}$  (reassignment of vectors)
10   for  $k \leftarrow 1$  to  $K$ 
11   do  $\vec{\mu}_k \leftarrow \frac{1}{|\omega_k|} \sum_{\vec{x} \in \omega_k} \vec{x}$  (recomputation of centroids)
12 return  $\{\vec{\mu}_1, \dots, \vec{\mu}_K\}$ 
```

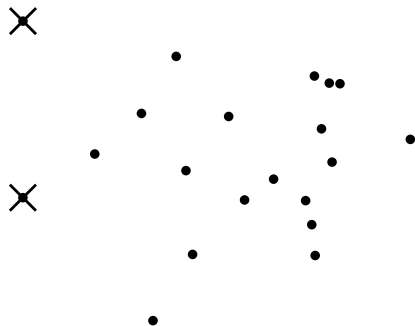
## Worked Example : Set of points to be clustered



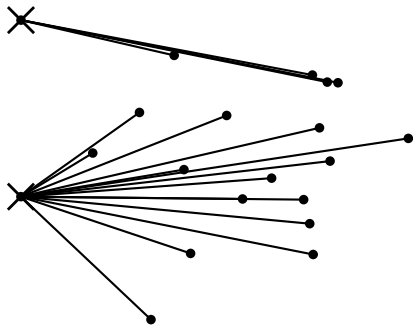
Exercise: (i) Guess what the

optimal clustering into two clusters is in this case; (ii) compute the centroids of the clusters □

## Worked Example: Random selection of initial centroids

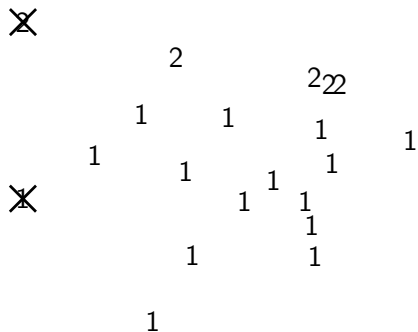


## Worked Example: Assign points to closest center



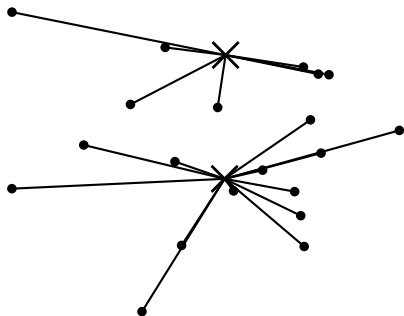


# Worked Example: Assignment

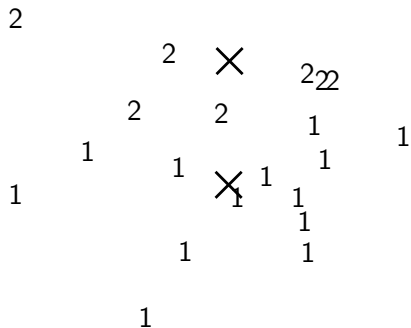




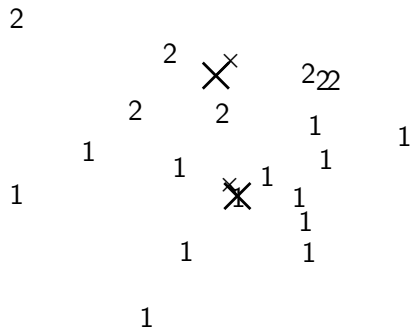
## Worked Example: Assign points to closest centroid



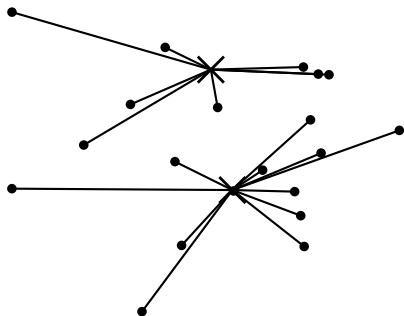
# Worked Example: Assignment



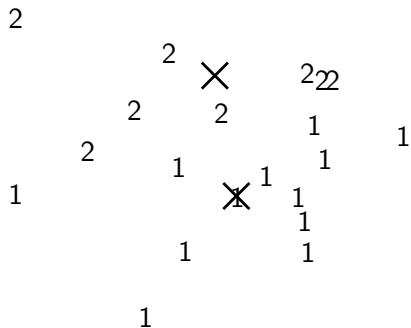
## Worked Example: Recompute cluster centroids



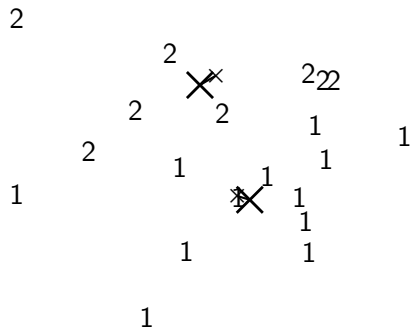
## Worked Example: Assign points to closest centroid



# Worked Example: Assignment

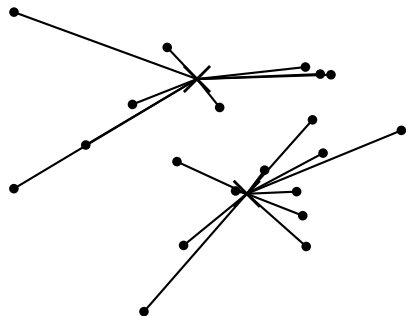


## Worked Example: Recompute cluster centroids





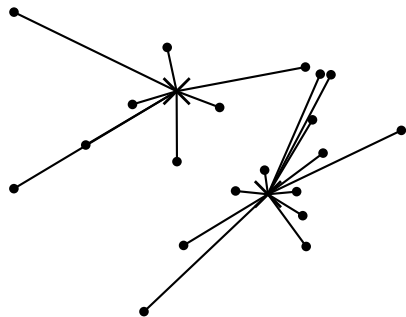
## Worked Example: Assign points to closest centroid



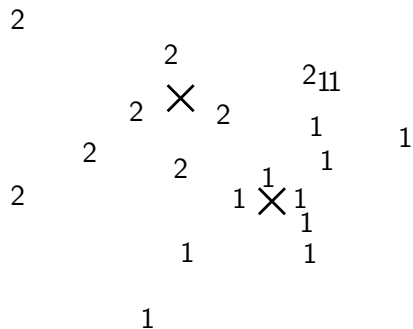




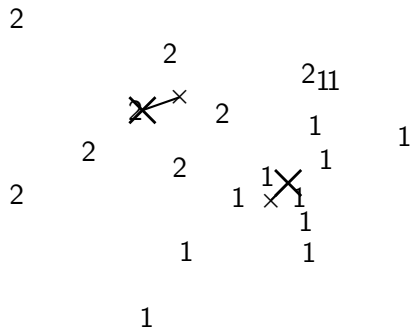
## Worked Example: Assign points to closest centroid



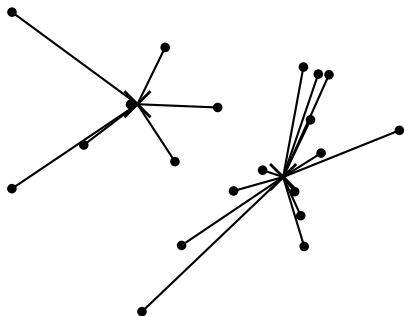
# Worked Example: Assignment



## Worked Example: Recompute cluster centroids



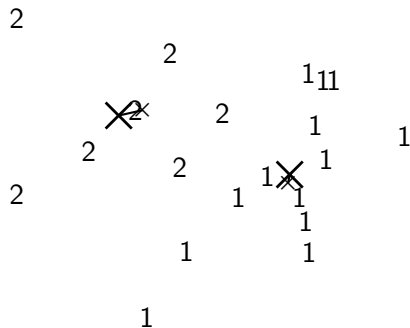
## Worked Example: Assign points to closest centroid



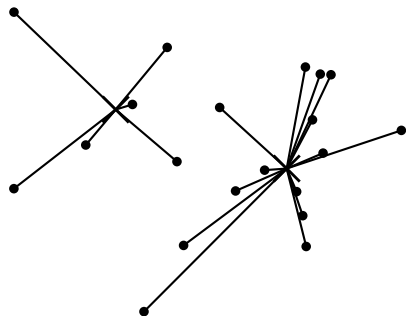




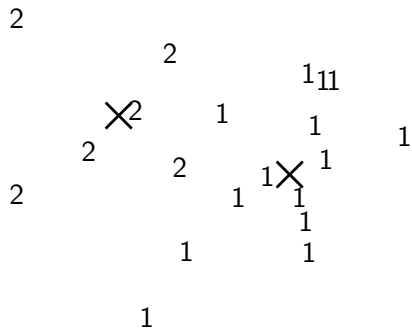
## Worked Example: Recompute cluster centroids



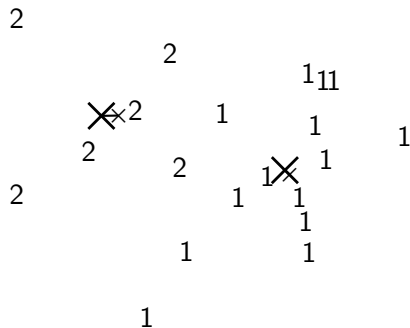
## Worked Example: Assign points to closest centroid



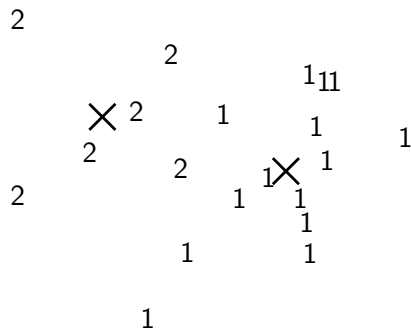
# Worked Example: Assignment



## Worked Example: Recompute cluster centroids



# Worked Ex.: Centroids and assignments after convergence



# $K$ -means is guaranteed to converge: Proof

- RSS = sum of all squared distances between document vector and closest centroid
- RSS decreases during each reassignment step.
  - because each vector is moved to a closer centroid
- RSS decreases during each recomputation step.
  - see next slide
- There is only a finite number of clusterings.
- Thus: We must reach a fixed point.
- Assumption: Ties are broken consistently.
- Finite set & monotonically decreasing  $\rightarrow$  convergence □

## Recomputation decreases average distance

$RSS = \sum_{k=1}^K RSS_k$  – the residual sum of squares (the “goodness” measure)

$$RSS_k(\vec{v}) = \sum_{\vec{x} \in \omega_k} \|\vec{v} - \vec{x}\|^2 = \sum_{\vec{x} \in \omega_k} \sum_{m=1}^M (v_m - x_m)^2$$
$$\frac{\partial RSS_k(\vec{v})}{\partial v_m} = \sum_{\vec{x} \in \omega_k} 2(v_m - x_m) = 0$$

$$v_m = \frac{1}{|\omega_k|} \sum_{\vec{x} \in \omega_k} x_m$$

The last line is the componentwise definition of the centroid! We minimize  $RSS_k$  when the old centroid is replaced with the new centroid.  $RSS$ , the sum of the  $RSS_k$ , must then also decrease during recomputation. □

# $K$ -means is guaranteed to converge

- But we don't know how long convergence will take!
- If we don't care about a few docs switching back and forth, then convergence is usually fast ( $< 10$ - $20$  iterations).
- However, complete convergence can take many more iterations.

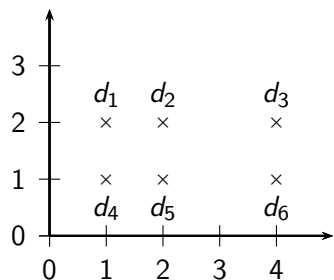




# Optimality of $K$ -means

- Convergence  $\neq$  optimality
- Convergence does not mean that we converge to the optimal clustering!
- This is the great weakness of  $K$ -means.
- If we start with a bad set of seeds, the resulting clustering can be horrible. □

## Exercise: Suboptimal clustering



- What is the optimal clustering for  $K = 2$ ?
- Do we converge on this clustering for arbitrary seeds  $d_i, d_j$ ?



# Initialization of $K$ -means

- Random seed selection is just one of many ways  $K$ -means can be initialized.
- Random seed selection is not very robust: It's easy to get a suboptimal clustering.
- Better ways of computing initial centroids:
  - Select seeds not randomly, but using some heuristic (e.g., filter out outliers or find a set of seeds that has “good coverage” of the document space)
  - Use hierarchical clustering to find good seeds
  - Select  $i$  (e.g.,  $i = 10$ ) different random sets of seeds, do a  $K$ -means clustering for each, select the clustering with lowest RSS



## Time complexity of $K$ -means

- Computing one distance of two vectors is  $O(M)$ .
- Reassignment step:  $O(KNM)$  (we need to compute  $KN$  document-centroid distances)
- Recomputation step:  $O(NM)$  (we need to add each of the document's  $< M$  values to one of the centroids)
- Assume number of iterations bounded by  $I$
- Overall complexity:  $O(IKNM)$  – linear in all important dimensions
- However: This is not a real worst-case analysis.
- In pathological cases, complexity can be worse than linear. □

# Outline

- 1 Recap
- 2 Clustering: Introduction
- 3 Clustering in IR
- 4  $K$ -means
- 5 Evaluation
- 6 How many clusters?

# What is a good clustering?

- Internal criteria
  - Example of an internal criterion: RSS in  $K$ -means
- But an internal criterion often does not evaluate the actual utility of a clustering in the application.
- Alternative: External criteria
  - Evaluate with respect to a human-defined classification □

## External criteria for clustering quality

- Based on a gold standard data set, e.g., the Reuters collection we also used for the evaluation of classification
- Goal: Clustering should reproduce the classes in the gold standard
- (But we only want to reproduce how documents are divided into groups, not the class labels.)
- First measure for how well we were able to reproduce the classes: **purity** □

## External criterion: Purity

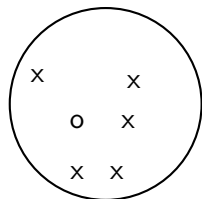
$$\text{purity}(\Omega, C) = \frac{1}{N} \sum_k \max_j |\omega_k \cap c_j|$$

- $\Omega = \{\omega_1, \omega_2, \dots, \omega_K\}$  is the set of clusters and  $C = \{c_1, c_2, \dots, c_J\}$  is the set of classes.
- For each cluster  $\omega_k$ : find class  $c_j$  with most members  $n_{kj}$  in  $\omega_k$
- Sum all  $n_{kj}$  and divide by total number of points □

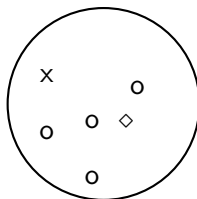


## Example for computing purity

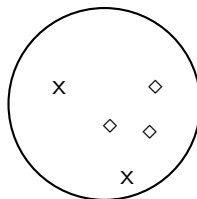
cluster 1



cluster 2



cluster 3



To compute

purity:  $5 = \max_j |\omega_1 \cap c_j|$  (class x, cluster 1);  $4 = \max_j |\omega_2 \cap c_j|$  (class o, cluster 2); and  $3 = \max_j |\omega_3 \cap c_j|$  (class  $\diamond$ , cluster 3).  
Purity is  $(1/17) \times (5 + 4 + 3) \approx 0.71$ . □

## Another external criterion: Rand index

- Purity can be increased easily by increasing  $K$  – a measure that does not have this problem: Rand index.

- Definition:  $RI = \frac{TP+TN}{TP+FP+FN+TN}$

- Based on 2x2 contingency table of all pairs of documents:

	same cluster	different clusters
same class	true positives (TP)	false negatives (FN)
different classes	false positives (FP)	true negatives (TN)

- $TP+FN+FP+TN$  is the total number of pairs.
- $TP+FN+FP+TN = \binom{N}{2}$  for  $N$  documents.
- Example:  $\binom{17}{2} = 136$  in o/◇/x example
- Each pair is either positive or negative (the clustering puts the two documents in the same or in different clusters) ...
- ...and either “true” (correct) or “false” (incorrect): the clustering decision is correct or incorrect. □

## Rand Index: Example

As an example, we compute RI for the o/◇/x example. We first compute TP + FP. The three clusters contain 6, 6, and 5 points, respectively, so the total number of “positives” or pairs of documents that are in the same cluster is:

$$TP + FP = \binom{6}{2} + \binom{6}{2} + \binom{5}{2} = 40$$

Of these, the x pairs in cluster 1, the o pairs in cluster 2, the ◇ pairs in cluster 3, and the x pair in cluster 3 are true positives:

$$TP = \binom{5}{2} + \binom{4}{2} + \binom{3}{2} + \binom{2}{2} = 20$$

Thus,  $FP = 40 - 20 = 20$ . FN and TN are computed similarly. □

## Rand measure for the o/◇/x example

	same cluster	different clusters	
same class	TP = 20	FN = 24	RI is then
different classes	FP = 20	TN = 72	

$$(20 + 72)/(20 + 20 + 24 + 72) \approx 0.68.$$



## Two other external evaluation measures

- Two other measures
- Normalized mutual information (NMI)
  - How much information does the clustering contain about the classification?
  - Singleton clusters (number of clusters = number of docs) have maximum MI
  - Therefore: normalize by entropy of clusters and classes
- F measure
  - Like Rand, but “precision” and “recall” can be weighted □

## Evaluation results for the o/◇/x example

	purity	NMI	RI	$F_5$	
lower bound	0.0	0.0	0.0	0.0	All four
maximum	1.0	1.0	1.0	1.0	
value for example	0.71	0.36	0.68	0.46	

measures range from 0 (really bad clustering) to 1 (perfect clustering).



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# How many clusters?

- Number of clusters  $K$  is given in many applications.
  - E.g., there may be an external constraint on  $K$ . Example: In the case of Scatter-Gather, it was hard to show more than 10–20 clusters on a monitor in the 90s.
- What if there is no external constraint? Is there a “right” number of clusters?
- One way to go: define an optimization criterion
  - Given docs, find  $K$  for which the optimum is reached.
  - What optimization criterion can we use?
  - We can't use RSS or average squared distance from centroid as criterion: always chooses  $K = N$  clusters. □



# Exercise

- Your job is to develop the clustering algorithms for a competitor to news.google.com
- You want to use  $K$ -means clustering.
- How would you determine  $K$ ?



## Simple objective function for $K$ : Basic idea

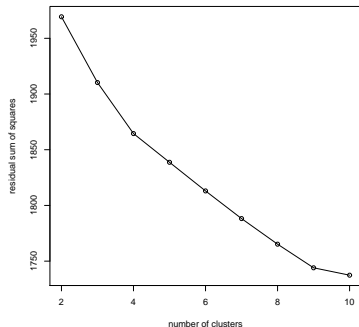
- Start with 1 cluster ( $K = 1$ )
- Keep adding clusters (= keep increasing  $K$ )
- Add a penalty for each new cluster
- Then trade off cluster penalties against average squared distance from centroid
- Choose the value of  $K$  with the best tradeoff



## Simple objective function for $K$ : Formalization

- Given a clustering, define the cost for a document as (squared) distance to centroid
- Define total **distortion**  $RSS(K)$  as sum of all individual document costs (corresponds to average distance)
- Then: penalize each cluster with a cost  $\lambda$
- Thus for a clustering with  $K$  clusters, total cluster penalty is  $K\lambda$
- Define the total cost of a clustering as distortion plus total cluster penalty:  $RSS(K) + K\lambda$
- Select  $K$  that minimizes  $(RSS(K) + K\lambda)$
- Still need to determine good value for  $\lambda \dots$  □

# Finding the “knee” in the curve



Pick the number of clusters where  
curve “flattens”. Here: 4 or 9. □

# Take-away today

- What is clustering?
- Applications of clustering in information retrieval
- $K$ -means algorithm
- Evaluation of clustering
- How many clusters?



# Resources

- Chapter 16 of IIR
- Resources at <http://cis1mu.org>
  - Keith van Rijsbergen on the cluster hypothesis (he was one of the originators)
  - Bing/Carrot2/Clusty: search result clustering systems
  - Stirling number: the number of distinct  $k$ -clusterings of  $n$  items