Feedforward Neural Networks and Word Embeddings

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WS 2023/2024
Outline

1. Linear models
2. Limitations of linear models
3. Neural networks
4. A neural language model
5. Word embeddings
Linear Models
Binary Classification with Linear Models

**Example:** the seminar at \(<\text{stime}>\) 4 pm will

**Classification task:** Do we have an \(<\text{stime}>\) tag in the current position?

<table>
<thead>
<tr>
<th>Word</th>
<th>Lemma</th>
<th>LexCat</th>
<th>Case</th>
<th>SemCat</th>
<th>Tag</th>
</tr>
</thead>
<tbody>
<tr>
<td>the</td>
<td>the</td>
<td>Art</td>
<td>low</td>
<td></td>
<td></td>
</tr>
<tr>
<td>seminar</td>
<td>seminar</td>
<td>Noun</td>
<td>low</td>
<td></td>
<td></td>
</tr>
<tr>
<td>at</td>
<td>at</td>
<td>Prep</td>
<td>low</td>
<td></td>
<td>stime</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>Digit</td>
<td>low</td>
<td></td>
<td></td>
</tr>
<tr>
<td>pm</td>
<td>pm</td>
<td>Other</td>
<td>low</td>
<td>timeid</td>
<td></td>
</tr>
<tr>
<td>will</td>
<td>will</td>
<td>Verb</td>
<td>low</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Feature Vector

Encode context into feature vector:

<table>
<thead>
<tr>
<th></th>
<th>bias term</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>-3_lemma_the</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>-3_lemma_giraffe</td>
<td>0</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>102</td>
<td>-2_lemma_seminar</td>
<td>1</td>
</tr>
<tr>
<td>103</td>
<td>-2_lemma_giraffe</td>
<td>0</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>202</td>
<td>-1_lemma_at</td>
<td>1</td>
</tr>
<tr>
<td>203</td>
<td>-1_lemma_giraffe</td>
<td>0</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>302</td>
<td>+1_lemma_4</td>
<td>1</td>
</tr>
<tr>
<td>303</td>
<td>+1_lemma_giraffe</td>
<td>0</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>
Dot product with weight vector

\[ h(X) = X \Theta^T = X \cdot \Theta \]

\[ X = \begin{bmatrix}
    x_0 = 1 \\
    x_1 = 1 \\
    x_2 = 0 \\
    \vdots \\
    x_{101} = 1 \\
    x_{102} = 0 \\
    \vdots \\
    x_{201} = 1 \\
    x_{202} = 0 \\
    \vdots \\
    x_{301} = 1 \\
    x_{302} = 0 \\
    \vdots
\end{bmatrix} \]

\[ \Theta = \begin{bmatrix}
    w_0 = 1.00 \\
    w_1 = 0.01 \\
    w_2 = 0.01 \\
    \vdots \\
    x_{101} = 0.01 \\
    x_{102} = 0.01 \\
    \vdots \\
    x_{201} = 0.01 \\
    x_{202} = 0.01 \\
    \vdots \\
    x_{301} = 0.01 \\
    x_{302} = 0.01 \\
    \vdots
\end{bmatrix} \]
Prediction with dot product

\[ h(X) = X \cdot \Theta \]

\[ = x_0 w_0 + x_1 w_1 + \cdots + x_n w_n \]

\[ = 1 \times 1 + 1 \times 0.01 + 0 \times 0.01 + \ldots + 0 \times 0.01 + 1 \times 0.01 \]
Example: the seminar at \(<\text{stime} \text{> 4 pm will}

Classification task: Do we have an \(<\text{stime} \text{> tag in the current position?}

Linear Model: \( h(X) = X \cdot \Theta \)

Prediction: If \( h(X) > 0 \), yes. Otherwise, no.
Getting the right weights

**Training:** Find weight vector $\Theta$ such that $h(X)$ is the correct answer as many times as possible.

→ Given a set $T$ of training examples $t_1, \cdots, t_n$ with correct labels $y_i$, find $\Theta$ such that $h(X(t_i)) = y_i$ for as many $t_i$ as possible.
→ $X(t_i)$ is the feature vector for the i-th training example $t_i$
Dot product with trained weight vector

\[ h(X) = X \cdot \Theta \]

\[ X = \begin{bmatrix}
    x_0 = 1 \\
    x_1 = 1 \\
    x_2 = 0 \\
    \vdots \\
    x_{101} = 1 \\
    x_{102} = 0 \\
    \vdots \\
    x_{201} = 1 \\
    x_{202} = 0 \\
    \vdots \\
    x_{301} = 1 \\
    x_{302} = 0 \\
    \vdots 
\end{bmatrix} \]

\[ \Theta = \begin{bmatrix}
    w_0 = 1.00 \\
    w_1 = 0.001 \\
    w_2 = 0.02 \\
    \vdots \\
    w_{101} = 0.012 \\
    w_{102} = 0.0015 \\
    \vdots \\
    w_{201} = 0.4 \\
    w_{202} = 0.005 \\
    \vdots \\
    w_{301} = 0.1 \\
    w_{302} = 0.04 \\
    \vdots 
\end{bmatrix} \]
Working with real-valued features

E.g. measure semantic similarity:

<table>
<thead>
<tr>
<th>Word</th>
<th>sim(time)</th>
</tr>
</thead>
<tbody>
<tr>
<td>the</td>
<td>0.0014</td>
</tr>
<tr>
<td>seminar</td>
<td>0.0014</td>
</tr>
<tr>
<td>at</td>
<td>0.1</td>
</tr>
<tr>
<td>4</td>
<td>2.01</td>
</tr>
<tr>
<td>pm</td>
<td>3.02</td>
</tr>
<tr>
<td>will</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Working with real-valued features

\[ h(X) = X \cdot \Theta \]

\[ X = \begin{bmatrix}
     x_0 = 1.0 \\
     x_1 = 50.5 \\
     x_2 = 52.2 \\
     \vdots \\
     x_{101} = 45.6 \\
     x_{102} = 60.9 \\
     \vdots \\
     x_{201} = 40.4 \\
     x_{202} = 51.9 \\
     \vdots \\
     x_{301} = 40.5 \\
     x_{302} = 35.8 \\
     \vdots 
\end{bmatrix} \]

\[ \Theta = \begin{bmatrix}
     w_0 = 1.00 \\
     w_1 = 0.001 \\
     w_2 = 0.02 \\
     \vdots \\
     x_{101} = 0.012 \\
     x_{102} = 0.0015 \\
     \vdots \\
     x_{201} = 0.4 \\
     x_{202} = 0.005 \\
     \vdots \\
     x_{301} = 0.1 \\
     x_{302} = 0.04 \\
     \vdots 
\end{bmatrix} \]
Working with real-valued features

\[ h(X) = X \cdot \Theta \]

\[ = x_0 w_0 + x_1 w_1 + \cdots + x_n w_n \]

\[ = 1.0 \times 1 + 50.5 \times 0.001 + \ldots + 40.5 \times 0.1 + 35.8 \times 0.04 \]

\[ = 540.5 \]
Working with real-valued features

**Classification task:** Do we have an `< stime >` tag in the current position?

**Prediction:** $h(X) = 540.5$

- Can we transform this into a probability?
Sigmoid function

We can push $h(X)$ between 0 and 1 using a **non-linear** activation function. The **sigmoid function** $\sigma(Z)$ is often used.
Classification task: Do we have an `< stime >` tag in the current position?

**Linear Model:** \( Z = X \cdot \Theta \)

**Prediction:** If \( Z > 0 \), yes. Otherwise, no.

**Logistic regression:**
- Use a **linear model** and squash values between 0 and 1.
  - Convert real values to probabilities
- Put threshold to 0.5.
- Positive class above threshold, negative class below.
Logistic Regression

\[ h(X) = \sigma(Z) \]
Linear Models: Limitations
Decision Boundary

What do **linear** models do?

- $\sigma(Z) > 0.5$ when $Z(= X \cdot \Theta) > 0$
- Model defines a **decision boundary** given by $X \cdot \Theta = 0$
  - positive examples (have stime tag)
  - negative examples (no stime tag)
Exercise

When we model a task with linear models, what assumption do we make about positive/negative examples?
Modeling 1: Learning a predictor for $\land$

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>a $\land$ b</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Features: $a$, $b$  
Feature values: binary

Can we learn a linear model to solve this problem?
Modeling 1: Learning a predictor for $\land$

\[ a \land b \]
Modeling 1: Logistic Regression

\[ x_0 \]
\[ x_1 \]
\[ x_2 \]

\[ z \]
\[ \sigma(Z) \]

\[
\begin{array}{c|c|c|c}
 x_0 & x_1 & x_2 & x_1 \land x_2 \\
\hline
 1 & 0 & 0 & \sigma(1 \times -30 + 0 \times 20 + 0 \times 20) = \sigma(-30) \approx 0 \\
 1 & 0 & 1 & \sigma(1 \times -30 + 0 \times 20 + 1 \times 20) = \sigma(-10) \approx 0 \\
 1 & 1 & 0 & \sigma(1 \times -30 + 1 \times 20 + 0 \times 20) = \sigma(-10) \approx 0 \\
 1 & 1 & 1 & \sigma(1 \times -30 + 1 \times 20 + 1 \times 20) = \sigma(10) \approx 1 \\
\end{array}
\]
Modeling 2: Learning a predictor for \textit{XNOR}

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>a \textit{XNOR} b</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Features : a, b                 Feature values : binary

Can we learn a linear model to solve this problem?
Non-linear decision boundaries

Can we learn a linear model to solve this problem?
Non-linear decision boundaries

Can we learn a linear model to solve this problem?
No! Decision boundary is **non-linear**.
Learning a predictor for \textit{XNOR}

Linear models not suited to learn non-linear decision boundaries. \textbf{Neural networks} can do that.
Neural Networks
Learning a predictor for \textit{XNOR}

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>a XNOR b</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Features : a, b Feature values : binary

Can we learn a \textbf{non-linear model} to solve this problem? Yes! E.g. through \textit{function composition}. 
Function Composition

\[
\sigma(Z)
\]

\[
\begin{array}{ccc}
\sigma(Z) & \sigma(Z) \\
20 & 20 \\
-30 & 10 \\
\end{array}
\]

\[
\begin{array}{ccc|c}
 x_0 & x_1 & x_2 & x_1 \land x_2 \\
 1 & 0 & 0 & \approx 0 \\
 1 & 0 & 1 & \approx 0 \\
 1 & 1 & 0 & \approx 0 \\
 1 & 1 & 1 & \approx 1 \\
\end{array}
\]

\[
\begin{array}{ccc|c}
 x_0 & x_1 & x_2 & \neg x_1 \land \neg x_2 \\
 1 & 0 & 0 & \approx 1 \\
 1 & 0 & 1 & \approx 0 \\
 1 & 1 & 0 & \approx 0 \\
 1 & 1 & 1 & \approx 0 \\
\end{array}
\]
Function Composition

\[ x_0, x_1, x_2 \]

\[
\begin{align*}
\sigma(Z_1) & \approx 0 & \sigma(Z_2) & \approx 1 \\
\sigma(Z_3) & = \sigma(1 \cdot -10 + 0 \cdot 20 + 1 \cdot 20) = \sigma(10) \approx 1 \\
\end{align*}
\]

\[
\begin{align*}
\sigma(Z_1) & \approx 0 & \sigma(Z_2) & \approx 0 \\
\sigma(Z_3) & = \sigma(1 \cdot -10 + 0 \cdot 20 + 0 \cdot 20) = \sigma(-10) \approx 0 \\
\end{align*}
\]

\[
\begin{align*}
\sigma(Z_1) & \approx 0 & \sigma(Z_2) & \approx 0 \\
\sigma(Z_3) & = \sigma(1 \cdot -10 + 0 \cdot 20 + 0 \cdot 20) = \sigma(-10) \approx 0 \\
\end{align*}
\]

\[
\begin{align*}
\sigma(Z_1) & \approx 1 & \sigma(Z_2) & \approx 0 \\
\sigma(Z_3) & = \sigma(1 \cdot -10 + 1 \cdot 20 + 0 \cdot 20) = \sigma(10) \approx 1 \\
\end{align*}
\]
Feedforward Neural Network

We just created a **feedforward neural network** with:

- 1 input layer $X$ (feature vector)
- 2 weight matrices $U = (\Theta_1, \Theta_2)$ and $V = \Theta_3$
- 1 hidden layer $H$ composed of:
  - 2 activations $A_1 = \sigma(Z_1)$ and $A_2 = \sigma(Z_2)$ where:
    - $Z_1 = X \cdot \Theta_1$
    - $Z_2 = X \cdot \Theta_2$
- 1 output unit $h(X) = \sigma(Z_3)$ where:
  - $Z_3 = H \cdot \Theta_3$
Feedforward Neural Network

Computation of hidden layer $H$:
- $A_1 = \sigma(X \cdot \Theta_1)$
- $A_2 = \sigma(X \cdot \Theta_2)$
- $B_0 = 1$ (bias term)

Computation of output unit $h(X)$:
- $h(X) = \sigma(H \cdot \Theta_3)$
General Feedforward Neural Network

Classification task: Do we have an \texttt{< stime >} tag in the current position?

Neural network: \[ h(X) = \sigma(H \cdot \Theta_n), \text{ with:} \]

\[ H = \begin{bmatrix} B_0 = 1 \\ A_1 = \sigma(X \cdot \Theta_1) \\ A_2 = \sigma(X \cdot \Theta_2) \\ \vdots \\ A_j = \sigma(X \cdot \Theta_j) \end{bmatrix} \]

Prediction: If \( h(X) > 0.5 \), yes. Otherwise, no.
Getting the right weights

**Training:** Find weight matrices $U = (\Theta_1, \Theta_2)$ and $V = \Theta_3$ such that $h(X)$ is the **correct answer** as many times as possible.

→ Given a set $T$ of training examples $t_1, \cdots t_n$ with **correct labels** $y_i$, find $U = (\Theta_1, \Theta_2)$ and $V = \Theta_3$ such that $h(X) = y_i$ for as many $t_i$ as possible.

→ Computation of $h(X)$ called **forward propagation**

→ Modify $U = (\Theta_1, \Theta_2)$ and $V = \Theta_3$ with error **back propagation**

The intuition behind back propagation is the same as the perceptron update!
Network architectures

Depending on task, a particular network architecture can be chosen:

Note: Bias terms omitted for simplicity
Multi-class classification

- More than two labels
- Instead of “yes” and “no”, predict $c_i \in C = \{c_1, \cdots, c_k\}$, where $k$ is the number of classes
- For instance, if we want to detect border tags for stime and etime, then we don’t only have the `<stime>` label but also: `</stime>`, `<etime>`, `</etime>`, no tag

- **Use 5 output units** (5 is the number of classes)
  - Output layer instead of a single output unit
  - The class with the highest activation is chosen
  - Probabilities can be obtained by dividing the exponentiated activation for a class by the sum of the exponentiated activations (“softmax”)
Summary: Neural Networks

- We showed how to use neural networks to solve non-linear decision problems.
- Neural networks are very powerful - much more powerful than linear models, even more powerful than decision trees.
- But we have been working with very simple features (binary features so far in our example).
- Neural networks can combine these simple features into very complex features (as was done previously with feature selection).
- But now we will show how neural language modeling led to the development of very powerful features, “word embeddings”, which are associated with word types.
A neural language model
Neural language model

- Early application of neural networks (Bengio et al. 2003)
- Task: Given $k$ previous words, predict the current word
  
  \[ P(w_t \mid w_{t-k}, \ldots, w_{t-2}, w_{t-1}) \]

- Previous (non-neural) approaches:

  Problem: Joint distribution of consecutive words difficult to obtain
  → chose small history to reduce complexity ($n=3$)
  → predict for unseen history through back-off to smaller history

  Drawbacks:
  
  Takes into account small context
  Does not model similarity between words
Word similarity for language modeling

1. The cat is walking in the bedroom
2. The dog was running in a room
3. A cat was running in a room
4. A dog was walking in a bedroom

→ Model similarity between (cat,dog), (room, bedroom)
→ Generalize from 1 to 2 etc.
Neural Language Model (LM)

Solution:
Use word embeddings to represent each word in history
→ Each word is represented in relation to the others
→ Distributed word feature vector
Feed to a neural network to learn parameters for the LM task
Training example: *The cat is walking in the bedroom*

**Neural network input:**
Look at words preceding *bedroom*
\[ \text{The cat is}_4 \text{ walking}_3 \text{ in}_2 \text{ the}_1 \text{ bedroom} \]
Create word embedding \((LT_i)\) for window
Give \(LT_i\) as input to Feedforward Neural Network

**Neural network training:**
Predict current word (forward propagation)
\[ \text{should be bedroom} \]
Train weights by backpropagating error
Feedforward Neural Network for LM

Input: word embeddings $LT_i$
Output: predicted label (current word)

Note: Bias terms omitted for simplicity
Feedforward Neural Network

**Input layer** \((X)\): Word features \(LT1, LT2, LT3, LT4\)

**Weight matrices** \(U, V\)

**Hidden layer** \((H)\): \(\sigma(X \cdot U + d)\)

**Output layer** \((0)\): \(H \cdot V + b\)

**Prediction**: \(h(X) = \text{softmax}(0)\)

- Predicted class is the one with highest probability (given by softmax)
Weight training

**Training**: Find weight matrices $U$ and $V$ such that $h(X)$ is the **correct answer** as many times as possible.

→ Given a set $T$ of training examples $t_1, \cdots t_n$ with **correct labels** $y_i$, find $U$ and $V$ such that $h(X) = y_i$ for as many $t_i$ as possible.

→ Computation of $h(X)$ with **forward propagation**

→ $U$ and $V$ with error **back propagation**
Forward Propagation

Forward propagation:

→ Perform all operations to get $h(X)$ from input $LT$. 

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Forward Propagation

Input layer ($X$): Word features LT1, LT2, LT3, LT4

Weight matrices $U$, $V$

Hidden layer ($H$): $\sigma(X \cdot U + d)$

Output layer ($\hat{0}$): $H \cdot V + b$

Prediction: $h(X) = \text{softmax}(\hat{0})$

- Predicted class is the one with highest probability (given by softmax)
Backpropagation

Goal of training: adjust weights such that correct label is predicted

→ Error between correct label and prediction is minimal

Sketch:

- Convert difference between prediction and error into derivatives
- Compute derivatives in each hidden layer from layer above
  - Backpropagate the error derivative with respect to the output of a unit
- Use derivatives with respect to the activations to get error derivatives with respect to incoming weights
Backpropagation:

→ Compute $E$
→ Compute $\frac{\partial E}{\partial O_i}$
Backpropagation

Compute **error at output E:**

Compare **output unit with** \( y^i \)

- \( y^i \) vector with 1 in correct class, 0 otherwise

\[
E = \frac{1}{n} \sum_{i=1}^{n} (y^i - O_i)^2 \quad \text{(mean squared)}
\]

Compute \( \frac{\partial E}{\partial O_i} \):

\[
\frac{\partial E}{\partial O_i} = -(y^i - O_i)
\]
Backpropagation:

→ Compute $\frac{\partial E}{\partial A_j}$
Backpropagation

Compute derivatives in each hidden layer from layer above:

- Compute derivative of error with respect to logit (output)
- Compute derivative of error with respect to previous hidden unit
- Compute derivative with respect to weights

→ Use recursion to do this for every layer
Backpropagation

\[
\frac{\partial E}{\partial w_{ij}}
\]

\[
E(h(X), y^i)
\]
Weight training

**Training:** Find weight matrices $U$ and $V$ such that $h(X)$ is the **correct answer** as many times as possible.

- Computation of $h(X)$ with **forward propagation**
- $U$ and $V$ with error **back propagation**

For each batch of training examples

1. Forward propagation to get predictions
2. Backpropagation of error
   - Gives gradient of $E$ given **input**
3. Modify weights (gradient descent)
4. Goto 1 until convergence
Word Embedding Layer

\[ E(h(X), y^i) \]

\[ input \quad U \quad hidden \quad V \quad output \]
Each word type encoded into index vector \( w_i = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \)

\( LT_i \) is dot product of weight matrix \( C \) with index of \( w_i \)

\( \rightarrow C \) is shared. Each column in \( C \) is used for all words (tokens) of a particular word-type.
Dot product with (trained) weight vector

\[ W = \{\text{the, cat, on, table, chair}\} \]

\[
\begin{bmatrix}
0 \\
0 \\
0 \\
1 \\
0
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
0.02 & 0.1 & 0.05 & 0.03 & 0.01 \\
0.15 & 0.2 & 0.01 & 0.02 & 0.11 \\
0.03 & 0.1 & 0.04 & 0.04 & 0.12
\end{bmatrix}
\]

\[
LT_{\text{table}} = w_{\text{table}} \cdot C = \begin{bmatrix}
0.03 \\
0.02 \\
0.04
\end{bmatrix}
\]

Words get mapped to lower dimension

\[ \rightarrow \text{Hyperparameter to be set} \]
Dot product with (initial) weight vector

\[ W = \{ \text{the, cat, on, table, chair} \} \]

\[
\begin{bmatrix}
0 \\
0 \\
0 \\
1 \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.01 & 0.01 & 0.01 & 0.01 & 0.01 \\
0.01 & 0.01 & 0.01 & 0.01 & 0.01 \\
0.01 & 0.01 & 0.01 & 0.01 & 0.01 \\
0.01 & 0.01 & 0.01 & 0.01 & 0.01 \\
0.01 & 0.01 & 0.01 & 0.01 & 0.01
\end{bmatrix}
\]

\[ LT_{\text{table}} = w_{\text{table}} \cdot C = \begin{bmatrix} 0.01 \\ 0.01 \\ 0.01 \end{bmatrix} \]

Feature vectors same for all words.
Feedforward Neural Network with Lookup Table

Note: Bias terms omitted for simplicity
Weight training

Training: Find weight matrices $C$, $U$ and $V$ such that $h(X)$ is the correct answer as many times as possible.

→ Given a set $T$ of training examples $t_1, \cdots t_n$ with correct labels $y_i$, find $C$, $U$ and $V$ such that $h(X) = y_i$ for as many $t_i$ as possible.
→ Computation of $h(X)$ with forward propagation
→ Modify $C$, $U$ and $V$ with error back propagation
Dot product with (trained) weight matrix

\[ W = \{ \text{the}, \text{cat}, \text{on}, \text{table}, \text{chair} \} \]

\[
\begin{bmatrix}
0 \\
0 \\
0 \\
1 \\
0
\end{bmatrix}
\]

\[
w_{\text{table}} = 
\begin{bmatrix}
0.02 & 0.1 & 0.05 & 0.03 & 0.01 \\
0.15 & 0.2 & 0.01 & 0.02 & 0.11 \\
0.03 & 0.1 & 0.04 & 0.04 & 0.12
\end{bmatrix}
\]

\[
LT_{\text{table}} = w_{\text{table}} \cdot C = 
\begin{bmatrix}
0.03 \\
0.02 \\
0.04
\end{bmatrix}
\]

Each word type gets a specific feature vector
Word Embeddings
Word Embeddings

- Representation of words in vector space
Word Embeddings

- Similar words are close to each other
  - Similarity is the cosine of the angle between two word vectors
Underlying thoughts

- Assume the equivalence of:
  - Two words are semantically similar.
  - Two words occur in similar contexts (Miller & Charles, roughly).
  - Two words have similar word neighbors in the corpus.

- Elements of this are from Leibniz, Harris, Firth, and Miller.
- Strictly speaking, similarity of neighbors is neither necessary nor sufficient for semantic similarity.
- But perhaps this is good enough.

*Adapted slide from Hinrich Schütze*
Learning word embeddings

Count-based methods:

- Compute cooccurrence statistics
- Learn high-dimensional representation
- Map sparse high-dimensional vectors to small dense representation
Word cooccurrence in Wikipedia

- corpus = English Wikipedia
- cooccurrence defined as occurrence within \( k = 10 \) words of each other
  - cooc.(rich,silver) = 186
  - cooc.(poor,silver) = 34
  - cooc.(rich,disease) = 17
  - cooc.(poor,disease) = 162
  - cooc.(rich,society) = 143
  - cooc.(poor,society) = 228

Adapted slide from Hinrich Schütze
Coocurrence-based Word Space

\[ \text{cooc.}(\text{poor}, \text{silver}) = 34, \text{cooc.}(\text{rich}, \text{silver}) = 186 \]
Coocurrence-based Word Space

\[ \text{cooc.}(\text{poor,disease}) = 162, \text{cooc.}(\text{rich,disease}) = 17. \]
Exercise

ccoc.(poor,society) = 228, cooc.(rich,society) = 143

How is it represented?
Coocurrence-based Word Space

\[ \text{cooc.}(\text{poor}, \text{society}) = 228, \text{cooc.}(\text{rich}, \text{society}) = 143 \]
Dimensionality of word space

- Up to now we’ve only used two dimension words: rich and poor.
- Do this for all possible words in a corpus → high-dimensional space
- Formally, there is no difference to a two-dimensional space with three vectors.

- Note: a word can have a dual role in word space.
  - Each word can, in principle, be a dimension word, an axis of the space.
  - But each word is also a vector in that space.

Adapted slide from Hinrich Schütze
Semantic similarity

Similarity is the cosine of the angle between two word vectors
Learning word embeddings

**Count-based** methods:
- Compute cooccurrence statistics
- Learn high-dimensional representation
- Map sparse high-dimensional vectors to small dense representation

**Neural networks:**
- Predict a word from its neighbors
- Learn (small) embedding vectors
Word vectors with Neural Networks

- **LM Task:** Given \( k \) previous words, predict the current word
  - For each word \( w \) in \( V \), model \( P(w_t|w_{t-1}, w_{t-2}, \ldots, w_{t-n}) \)
  - **Learn embeddings** \( C \) of words

- **Word embeddings learning task:** Given \( k \) context words, predict the current word
  - **Learn embeddings** \( C \) of words
Network architecture

Given words $w_{t-2}, w_{t-1}, w_{t+1}$ and $w_{t+2}$, predict $w_t$ ("CBOW")

Note: Bias terms omitted for simplicity
Network architecture

We want the context vectors $\rightarrow$ embed words in shared space
Note: Bias terms omitted for simplicity
Getting the Word Embeddings

Note: Bias terms omitted for simplicity
Simplifications

- Remove hidden layer
- Sum over all projections
Simplifications

Remove hidden layer and sum over context
Note: Bias terms omitted for simplicity
Simplifications

- Single logistic unit instead of output layer
  - No need for distribution over words (only vector representation)
  - Task as binary classification problem:
    - Given input and weight matrix say if \( w_t \) is current word
    - We know the correct \( w_t \), how do we get the wrong ones?
      - negative sampling
Word2Vec

- BOW model (Mikolov. 2013)
- Skip-gram model:
  - Input is $w_t$
  - Prediction is $w_{t+2}, w_{t+1}, w_{t-1}$ and $w_{t-2}$
Applications

Semantic similarity:

- How similar are the words:
  - coast and shore; rich and money; happiness and disease; close and open
- WordSim-353 (Finkelstein et al. 2002)
  - Measure associations
- SimLex-999
  - Only measure semantic similarity

Other tasks:

- Use word embeddings as input features for other tasks (e.g. sentiment analysis, language modeling, named entity recognition)
Recap

- Cannot fit data with non-linear decision boundary with linear models

**Solution**: compose non-linear functions with neural networks

→ Successful in many NLP applications:
  - Language modeling
  - Learning word embeddings

- Feeding word embeddings into neural networks has proven successful in many NLP tasks, e.g.:
  - Sentiment Analysis
  - Named Entity Recognition
Questions?
Some Further Issues

- The backup slides (at the end) show the details of backpropagation, it is a good idea to look at these.
- Neural networks can be shown to approximate any function arbitrarily well. See the intuitive discussion of this property in this online book, in chapter 4:
- I also highly recommend the other chapters in this book!
Thank you for your attention.
The next slide shows the actual computation of backpropagation, showing the derivatives that are computed. The actual updates are also shown, these are more intuitive than the derivatives for many people.
Backpropagation

Compute **derivatives** in each hidden layer from layer above:

Compute derivative of error with respect to logit (output)
\[
\frac{\partial E}{\partial Z_i} = \frac{\partial E}{\partial O_i} \frac{\partial O_i}{\partial Z_i} = \frac{\partial E}{\partial O_i} O_i (1 - O_i) \quad \text{(Note: } O_i = \frac{1}{1 + e^{-Z_i}})\]

Compute derivative of error with respect to previous hidden unit
\[
\frac{\partial E}{\partial A_j} = \sum_i \frac{\partial Z_i}{\partial A_j} \frac{\partial E}{\partial Z_i} = \sum_i w_{ji} \frac{\partial E}{\partial Z_i}\]

Compute derivative with respect to weights
\[
\frac{\partial E}{\partial w_{ji}} = \frac{\partial Z_i}{\partial w_{ji}} \frac{\partial E}{\partial Z_i} = O_i \frac{\partial E}{\partial Z_i}\]

→ Use **recursion** to do this for every layer
Backpropagation

\[ \frac{\partial E}{\partial w_{ij}} \]

\[ \frac{\partial E}{\partial w_{ij}} \]

\[ E(h(X), y^i) \]

\[ \frac{\partial E}{\partial w_{ij}} \]

\[ \frac{\partial E}{\partial w_{ij}} \]
Weight training

Training: Find weight matrices $U$ and $V$ such that $h(X)$ is the correct answer as many times as possible.

→ Computation of $h(X)$ with forward propagation
→ $U$ and $V$ with error back propagation

For each batch of training examples
1. Forward propagation to get predictions
2. Backpropagation of error
   ▶ Gives gradient of $E$ given input
3. Modify weights (gradient descent)
4. Goto 1 until convergence