Feedforward Neural Networks and Word Embeddings

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(slides originally by Dr. Fabienne Braune)

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Outline

1. Linear models
2. Limitations of linear models
3. Neural networks
4. A neural language model
5. Word embeddings
Linear Models
Example: the seminar at <stime> 4 pm will

Classification task: Do we have an <stime> tag in the current position?

<table>
<thead>
<tr>
<th>Word</th>
<th>Lemma</th>
<th>LexCat</th>
<th>Case</th>
<th>SemCat</th>
<th>Tag</th>
</tr>
</thead>
<tbody>
<tr>
<td>the</td>
<td>the</td>
<td>Art</td>
<td>low</td>
<td></td>
<td></td>
</tr>
<tr>
<td>seminar</td>
<td>seminar</td>
<td>Noun</td>
<td>low</td>
<td></td>
<td></td>
</tr>
<tr>
<td>at</td>
<td>at</td>
<td>Prep</td>
<td>low</td>
<td></td>
<td>stime</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>Digit</td>
<td>low</td>
<td></td>
<td></td>
</tr>
<tr>
<td>pm</td>
<td>pm</td>
<td>Other</td>
<td>low</td>
<td>timeid</td>
<td></td>
</tr>
<tr>
<td>will</td>
<td>will</td>
<td>Verb</td>
<td>low</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Feature Vector

Encode context into *feature vector*:

<table>
<thead>
<tr>
<th></th>
<th>bias term</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-3_lemma_the</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>-3_lemma_giraffe</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>102</td>
<td>-2_lemma_seminar</td>
<td>1</td>
</tr>
<tr>
<td>103</td>
<td>-2_lemma_giraffe</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>202</td>
<td>-1_lemma_at</td>
<td>1</td>
</tr>
<tr>
<td>203</td>
<td>-1_lemma_giraffe</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>302</td>
<td>+1_lemma_4</td>
<td>1</td>
</tr>
<tr>
<td>303</td>
<td>+1_lemma_giraffe</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>
Dot product with weight vector

\[ h(X) = X\Theta^T = X \cdot \Theta \]

\[
X = \begin{bmatrix}
x_0 = 1 \\
x_1 = 1 \\
x_2 = 0 \\
\ldots \\
x_{101} = 1 \\
x_{102} = 0 \\
\ldots \\
x_{201} = 1 \\
x_{202} = 0 \\
\ldots \\
x_{301} = 1 \\
x_{302} = 0 \\
\ldots 
\end{bmatrix}
\]

\[
\Theta = \begin{bmatrix}
w_0 = 1.00 \\
w_1 = 0.01 \\
w_2 = 0.01 \\
\ldots \\
x_{101} = 0.01 \\
x_{102} = 0.01 \\
\ldots \\
x_{201} = 0.01 \\
x_{202} = 0.01 \\
\ldots \\
x_{301} = 0.01 \\
x_{302} = 0.01 \\
\ldots 
\end{bmatrix}
\]
Prediction with dot product

\[ h(X) = X \cdot \Theta \]
\[ = x_0 w_0 + x_1 w_1 + \cdots + x_n w_n \]
\[ = 1 \times 1 + 1 \times 0.01 + 0 \times 0.01 + \ldots + 0 \times 0.01 + 1 \times 0.01 \]
Predictions with linear models

Example: the seminar at \(<\text{stime}\)> 4 pm will

Classification task: Do we have an \(<\text{stime}\)> tag in the current position?

Linear Model: \( h(X) = X \cdot \Theta \)

Prediction: If \( h(X) > 0 \), yes. Otherwise, no.
Getting the right weights

**Training:** Find weight vector $\Theta$ such that $h(X)$ is the **correct answer** as many times as possible.

$\rightarrow$ Given a set $T$ of training examples $t_1, \cdots t_n$ with **correct labels** $y_i$, find $\Theta$ such that $h(X(t_i)) = y_i$ for as many $t_i$ as possible.

$\rightarrow$ $X(t_i)$ is the feature vector for the i-th training example $t_i$
Dot product with trained weight vector

\[ h(X) = X \cdot \Theta \]

\[ X = \begin{bmatrix}
    x_0 &= 1 \\
    x_1 &= 1 \\
    x_2 &= 0 \\
    \vdots \\
    x_{101} &= 1 \\
    x_{102} &= 0 \\
    \vdots \\
    x_{201} &= 1 \\
    x_{202} &= 0 \\
    \vdots \\
    x_{301} &= 1 \\
    x_{302} &= 0 \\
    \vdots 
\end{bmatrix} \]

\[ \Theta = \begin{bmatrix}
    w_0 &= 1.00 \\
    w_1 &= 0.001 \\
    w_2 &= 0.02 \\
    \vdots \\
    w_{101} &= 0.012 \\
    w_{102} &= 0.0015 \\
    \vdots \\
    w_{201} &= 0.4 \\
    w_{202} &= 0.005 \\
    \vdots \\
    w_{301} &= 0.1 \\
    w_{302} &= 0.04 \\
    \vdots 
\end{bmatrix} \]
Working with real-valued features

E.g. measure semantic similarity:

<table>
<thead>
<tr>
<th>Word</th>
<th>sim(time)</th>
</tr>
</thead>
<tbody>
<tr>
<td>the</td>
<td>0.0014</td>
</tr>
<tr>
<td>seminar</td>
<td>0.0014</td>
</tr>
<tr>
<td>at</td>
<td>0.1</td>
</tr>
<tr>
<td>4</td>
<td>2.01</td>
</tr>
<tr>
<td>pm</td>
<td>3.02</td>
</tr>
<tr>
<td>will</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Working with real-valued features

\[ h(X) = X \cdot \Theta \]

\[ X = \begin{bmatrix}
    x_0 = 1.0 \\
    x_1 = 50.5 \\
    x_2 = 52.2 \\
    \vdots \\
    x_{101} = 45.6 \\
    x_{102} = 60.9 \\
    \vdots \\
    x_{201} = 40.4 \\
    x_{202} = 51.9 \\
    \vdots \\
    x_{301} = 40.5 \\
    x_{302} = 35.8 \\
    \vdots 
\end{bmatrix} \]

\[ \Theta = \begin{bmatrix}
    w_0 = 1.00 \\
    w_1 = 0.001 \\
    w_2 = 0.02 \\
    \vdots \\
    x_{101} = 0.012 \\
    x_{102} = 0.0015 \\
    \vdots \\
    x_{201} = 0.4 \\
    x_{202} = 0.005 \\
    \vdots \\
    x_{301} = 0.1 \\
    x_{302} = 0.04 \\
    \vdots 
\end{bmatrix} \]
Working with real-valued features

\[ h(X) = X \cdot \Theta \]
\[ = x_0 w_0 + x_1 w_1 + \cdots + x_n w_n \]
\[ = 1.0 \times 1 + 50.5 \times 0.001 + \cdots + 40.5 \times 0.1 + 35.8 \times 0.04 \]
\[ = 540.5 \]
Working with real-valued features

Classification task: Do we have an `<stime>` tag in the current position?
Prediction: $h(X) = 540.5$

Can we transform this into a probability?
Sigmoid function

We can push $h(X)$ between 0 and 1 using a non-linear activation function. The **sigmoid function** $\sigma(Z)$ is often used.
Logistic Regression

Classification task: Do we have an `<stime>` tag in the current position?

Linear Model: $Z = X \cdot \Theta$

Prediction: If $Z > 0$, yes. Otherwise, no.

Logistic regression:
- Use a **linear model** and squash values between 0 and 1.
  - Convert real values to probabilities
- Put threshold to 0.5.
- Positive class above threshold, negative class below.
Logistic Regression

\[ h(X) = \sigma(Z) \]
Linear Models: Limitations
Decision Boundary

What do linear models do?

- \( \sigma(Z) > 0.5 \) when \( Z = X \cdot \Theta \) > 0
- Model defines a decision boundary given by \( X \cdot \Theta = 0 \)
  - positive examples (have stime tag)
  - negative examples (no stime tag)
Exercise

When we model a task with linear models, what assumption do we make about positive/negative examples?
Modeling 1: Learning a predictor for $\land$

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>$a \land b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Features: $a, b$  
Feature values: binary

Can we learn a linear model to solve this problem?
Modeling 1: Learning a predictor for $\land$
Modeling 1: Logistic Regression

\[ x_0 \]
\[ x_1 \]
\[ x_2 \]

\[ z = \sigma(Z) \]

<table>
<thead>
<tr>
<th>( x_0 )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_1 \wedge x_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>( \sigma(1 \cdot -30 + 0 \cdot 20 + 0 \cdot 20) = \sigma(-30) \approx 0 )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>( \sigma(1 \cdot -30 + 0 \cdot 20 + 1 \cdot 20) = \sigma(-10) \approx 0 )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>( \sigma(1 \cdot -30 + 1 \cdot 20 + 0 \cdot 20) = \sigma(-10) \approx 0 )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>( \sigma(1 \cdot -30 + 1 \cdot 20 + 1 \cdot 20) = \sigma(10) \approx 1 )</td>
</tr>
</tbody>
</table>
Modeling 2: Learning a predictor for \textit{XNOR}

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>a XNOR b</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Features : a, b  
Feature values : binary

Can we learn a linear model to solve this problem?
Non-linear decision boundaries

Can we learn a linear model to solve this problem?
Non-linear decision boundaries

Can we learn a linear model to solve this problem?
No! Decision boundary is **non-linear**.
Learning a predictor for **XNOR**

Linear models not suited to learn non-linear decision boundaries. **Neural networks** can do that.
Neural Networks
Learning a predictor for $XNOR$

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>$a \ XNOR \ b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Features: $a, b$  
Feature values: binary

Can we learn a **non-linear model** to solve this problem?  
Yes! E.g. through **function composition**.
Function Composition

\[ x_0, x_1, x_2 \]

\[ \sigma(Z) \]

\[ \begin{array}{cccc|c}
   x_0 & x_1 & x_2 & x_1 \land x_2 \\
   \hline
   1 & 0 & 0 & \approx 0 \\
   1 & 0 & 1 & \approx 0 \\
   1 & 1 & 0 & \approx 0 \\
   1 & 1 & 1 & \approx 1 \\
\end{array} \]

\[ \begin{array}{cccc|c}
   x_0 & x_1 & x_2 & \neg x_1 \land \neg x_2 \\
   \hline
   1 & 0 & 0 & \approx 1 \\
   1 & 0 & 1 & \approx 0 \\
   1 & 1 & 0 & \approx 0 \\
   1 & 1 & 1 & \approx 0 \\
\end{array} \]
Function Composition

\[ x_0 \rightarrow Z_0 \rightarrow Z_1 \rightarrow Z_2 \rightarrow Z_3 \]

\[ x_0, x_1, x_2 \rightarrow \sigma(Z_1) \rightarrow \sigma(Z_2) \rightarrow \sigma(Z_3) \]

<table>
<thead>
<tr>
<th>(x_0)</th>
<th>(x_1)</th>
<th>(x_2)</th>
<th>(\sigma(Z_1))</th>
<th>(\sigma(Z_2))</th>
<th>(\sigma(Z_3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>(\approx 0)</td>
<td>(\approx 1)</td>
<td>(\sigma(1 \times -10 + 0 \times 20 + 1 \times 20) = \sigma(10) \approx 1)</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>(\approx 0)</td>
<td>(\approx 0)</td>
<td>(\sigma(1 \times -10 + 0 \times 20 + 0 \times 20) = \sigma(-10) \approx 0)</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>(\approx 0)</td>
<td>(\approx 0)</td>
<td>(\sigma(1 \times -10 + 0 \times 20 + 0 \times 20) = \sigma(-10) \approx 0)</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>(\approx 1)</td>
<td>(\approx 0)</td>
<td>(\sigma(1 \times -10 + 1 \times 20 + 0 \times 20) = \sigma(10) \approx 1)</td>
</tr>
</tbody>
</table>
Feedforward Neural Network

We just created a **feedforward neural network** with:

- 1 input layer $X$ (feature vector)
- 2 weight matrices $U = (\Theta_1, \Theta_2)$ and $V = \Theta_3$
- 1 hidden layer $H$ composed of:
  - 2 activations $A_1 = \sigma(Z_1)$ and $A_2 = \sigma(Z_2)$ where:
    - $Z_1 = X \cdot \Theta_1$
    - $Z_2 = X \cdot \Theta_2$
- 1 output unit $h(X) = \sigma(Z_3)$ where:
  - $Z_3 = H \cdot \Theta_3$
Feedforward Neural Network

Computation of hidden layer $\mathbf{H}$:
- $A_1 = \sigma(X \cdot \Theta_1)$
- $A_2 = \sigma(X \cdot \Theta_2)$
- $B_0 = 1$ (bias term)

Computation of output unit $h(X)$:
- $h(X) = \sigma(\mathbf{H} \cdot \Theta_3)$
General Feedforward Neural Network

Classification task: Do we have an $<$ stime $>$ tag in the current position?

Neural network: $h(X) = \sigma(H \cdot \Theta_n)$, with:

$$H = \begin{bmatrix} B_0 = 1 \\ A_1 = \sigma(X \cdot \Theta_1) \\ A_2 = \sigma(X \cdot \Theta_2) \\ \vdots \\ A_j = \sigma(X \cdot \Theta_j) \end{bmatrix}$$

Prediction: If $h(X) > 0.5$, yes. Otherwise, no.
Getting the right weights

**Training:** Find weight matrices \( U = (\Theta_1, \Theta_2) \) and \( V = \Theta_3 \) such that \( h(X) \) is the *correct answer* as many times as possible.

\[ \rightarrow \text{Given a set } T \text{ of training examples } t_1, \cdots, t_n \text{ with correct labels } y_i, \text{ find } U = (\Theta_1, \Theta_2) \text{ and } V = \Theta_3 \text{ such that } h(X) = y_i \text{ for as many } t_i \text{ as possible.} \]

\[ \rightarrow \text{Computation of } h(X) \text{ called forward propagation} \]

\[ \rightarrow \text{Modify } U = (\Theta_1, \Theta_2) \text{ and } V = \Theta_3 \text{ with error back propagation} \]

The intuition behind back propagation is the same as the perceptron update!
Network architectures

Depending on task, a particular network architecture can be chosen:

![Diagram of a neural network with input layers, hidden layers, and output layers.]

Note: Bias terms omitted for simplicity
Multi-class classification

- More than two labels
- Instead of “yes” and “no”, predict \( c_i \in C = \{c_1, \ldots, c_k\} \), where \( k \) is the number of classes
- For instance, if we want to detect border tags for stime and etime, then we don’t only have the \( <\text{stime}> \) label but also: \( </\text{stime}> \), \( <\text{etime}> \), \( </\text{etime}> \), no tag

- **Use 5 output units** (5 is the number of classes)
  - Output layer instead of a single output unit
  - The class with the highest activation is chosen
  - Probabilities can be obtained by dividing the exponentiated activation for a class by the sum of the exponentiated activations (“softmax”)
Summary: Neural Networks

- We showed how to use neural networks to solve non-linear decision problems.
- Neural networks are very powerful - much more powerful than linear models, even more powerful than decision trees.
- But we have been working with very simple features (binary features so far in our example).
- Neural networks can combine these simple features into very complex features (as was done previously with feature selection).
- But now we will show how neural language modeling led to the development of very powerful features, “word embeddings”, which are associated with word types.
A neural language model
Neural language model

- Early application of neural networks (Bengio et al. 2003)
- Task: Given $k$ previous words, predict the current word
  
  Estimate: $P(w_t|w_{t-k}, \ldots, w_{t-2}, w_{t-1})$

- Previous (non-neural) approaches:
  
  **Problem:** Joint distribution of consecutive words difficult to obtain
  → chose small history to reduce complexity ($n=3$)
  → predict for unseen history through back-off to smaller history

  **Drawbacks:**
  
  Takes into account small context
  **Does not model similarity between words**
Word similarity for language modeling

1. The cat is walking in the bedroom
2. The dog was running in a room
3. A cat was running in a room
4. A dog was walking in a bedroom

→ Model similarity between (cat, dog), (room, bedroom)
→ Generalize from 1 to 2 etc.
Neural Language Model (LM)

Solution:
Use word embeddings to represent each word in history
→ Each word is represented in relation to the others
→ Distributed word feature vector
Feed to a neural network to learn parameters for the LM task
Feedforward Neural Network for LM

Training example: *The cat is walking in the bedroom*

Neural network input:

Look at words preceding *bedroom*

→ *The cat is walking in the bedroom*

→ Create word embedding (*LT*\(_i\)) for window

Give *LT*\(_i\) as input to Feedforward Neural Network

Neural network training:

Predict current word (forward propagation)

→ should be *bedroom*

Train weights by backpropagating error
Feedforward Neural Network for LM

Input: word embeddings $LT_i$
Output: predicted label (current word)

Note: Bias terms omitted for simplicity
Feedforward Neural Network

Input layer \((X)\): Word features LT1, LT2, LT3, LT4

Weight matrices \(U, V\)

Hidden layer \((H)\): \(\sigma(X \cdot U + d)\)

Output layer \((0)\): \(H \cdot V + b\)

Prediction: \(h(X) = \text{softmax}(0)\)

- Predicted class is the one with highest probability (given by softmax)
Weight training

Training: Find weight matrices $U$ and $V$ such that $h(X)$ is the correct answer as many times as possible.

→ Given a set $T$ of training examples $t_1, \cdots t_n$ with correct labels $y_i$, find $U$ and $V$ such that $h(X) = y_i$ for as many $t_i$ as possible.

→ Computation of $h(X)$ with forward propagation

→ $U$ and $V$ with error back propagation
Forward Propagation

Forward propagation:

→ Perform all operations to get $h(X)$ from input $LT$. 

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Forward Propagation

**Input layer** \((X)\): Word features \(LT1, LT2, LT3, LT4\)

**Weight matrices** \(U, V\)

**Hidden layer** \((H)\): \(\sigma(X \cdot U + d)\)

**Output layer** \((O)\): \(H \cdot V + b\)

**Prediction**: \(h(X) = \text{softmax}(O)\)

- Predicted class is the one with highest probability (given by softmax)
Backpropagation

Goal of training: adjust weights such that correct label is predicted

→ Error between correct label and prediction is minimal

Sketch:

- Convert difference between prediction and error into derivatives
- Compute derivatives in each hidden layer from layer above
  - Backpropagate the error derivative with respect to the output of a unit
- Use derivatives with respect to the activations to get error derivatives with respect to incoming weights
Backpropagation:

→ Compute $E$

→ Compute $\frac{\partial E}{\partial O_i}$
Backpropagation

Compute error at output $E$:

- Compare output unit with $y^i$
  - $y^i$ vector with 1 in correct class, 0 otherwise

$$E = \frac{1}{n} \sum_{i=1}^{n} (y_i - O_i)^2 \text{ (mean squared)}$$

Compute $\frac{\partial E}{\partial O_i}$:

$$\frac{\partial E}{\partial O_i} = -(y_i - O_i)$$
Backpropagation:

→ Compute $\frac{\partial E}{\partial A_j}$
Backpropagation

Compute **derivatives** in each hidden layer from layer above:

- Compute derivative of **error** with respect to logit (output)
- Compute derivative of **error** with respect to previous hidden unit
- Compute derivative with respect to **weights**

→ Use **recursion** to do this for every layer
Backpropagation

\[ \frac{\partial E}{\partial w_{ij}} \]

\[ \frac{\partial E}{\partial w_{ij}} \]

\[ E(h(X), y^i) \]

\[ \text{input} \quad U \quad \frac{\partial E}{\partial A_j} \quad V \quad \frac{\partial E}{\partial O_i} \]
Weight training

Training: Find weight matrices $U$ and $V$ such that $h(X)$ is the correct answer as many times as possible.

$→$ Computation of $h(X)$ with forward propagation
$→$ $U$ and $V$ with error back propagation

For each batch of training examples

1. Forward propagation to get predictions
2. Backpropagation of error
   - Gives gradient of $E$ given input
3. Modify weights (gradient descent)
4. Goto 1 until convergence
Word Embedding Layer

\[ E(h(X), y^i) \]

\[ \text{input} \quad U \quad \text{hidden} \quad V \quad \text{output} \]
Word Embedding Layer

- Each word type encoded into index vector \( w_i \) =
  \[
  \begin{bmatrix}
  0 \\
  1 \\
  0 \\
  0 \\
  0
  \end{bmatrix}
  \]

- \( LT_i \) is dot product of weight matrix \( C \) with index of \( w_i \)
  \[\rightarrow C \text{ is shared. Each column in C is used for all words (tokens) of a particular word-type.} \]
Dot product with (trained) weight vector

\[ W = \{ \text{the, cat, on, table, chair} \} \]

\[
\begin{bmatrix}
0 \\
0 \\
0 \\
1 \\
0
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
0.02 & 0.1 & 0.05 & 0.03 & 0.01 \\
0.15 & 0.2 & 0.01 & 0.02 & 0.11 \\
0.03 & 0.1 & 0.04 & 0.04 & 0.12
\end{bmatrix}
\]

\[
LT_{\text{table}} = w_{\text{table}} \cdot C = \begin{bmatrix}
0.03 \\
0.02 \\
0.04
\end{bmatrix}
\]

Words get mapped to lower dimension
→ Hyperparameter to be set
Dot product with (initial) weight vector

\[ W = \{ \text{the, cat, on, table, chair} \} \]

\[
\begin{bmatrix}
0 \\
0 \\
0 \\
1 \\
0
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
0.01 & 0.01 & 0.01 & 0.01 & 0.01 \\
0.01 & 0.01 & 0.01 & 0.01 & 0.01 \\
0.01 & 0.01 & 0.01 & 0.01 & 0.01 \\
0.01 & 0.01 & 0.01 & 0.01 & 0.01 \\
0.01 & 0.01 & 0.01 & 0.01 & 0.01
\end{bmatrix}
\]

\[
LT_{\text{table}} = w_{\text{table}} \cdot C = \begin{bmatrix}
0.01 \\
0.01 \\
0.01
\end{bmatrix}
\]

Feature vectors same for all words.
Feedforward Neural Network with Lookup Table

Note: Bias terms omitted for simplicity
Weight training

Training: Find weight matrices $C$, $U$ and $V$ such that $h(X)$ is the correct answer as many times as possible.

→ Given a set $T$ of training examples $t_1, \cdots t_n$ with correct labels $y_i$, find $C$, $U$ and $V$ such that $h(X) = y_i$ for as many $t_i$ as possible.
→ Computation of $h(X)$ with forward propagation
→ Modify $C$, $U$ and $V$ with error back propagation
Dot product with (trained) weight matrix

\[ W = \{ \text{the, cat, on, table, chair} \} \]

\[
\begin{bmatrix}
0 \\
0 \\
0 \\
1 \\
0
\end{bmatrix}
\quad C =
\begin{bmatrix}
0.02 & 0.1 & 0.05 & 0.03 & 0.01 \\
0.15 & 0.2 & 0.01 & 0.02 & 0.11 \\
0.03 & 0.1 & 0.04 & 0.04 & 0.12
\end{bmatrix}
\]

\[ LT_{\text{table}} = w_{\text{table}} \cdot C =
\begin{bmatrix}
0.03 \\
0.02 \\
0.04
\end{bmatrix}
\]

Each word type gets a **specific** feature vector
WORD EMBEDDINGS
Word Embeddings

- Representation of words in vector space
Word Embeddings

- Similar words are close to each other
  → Similarity is the cosine of the angle between two word vectors
Underlying thoughts

- Assume the equivalence of:
  - Two words are semantically similar.
  - Two words occur in similar contexts (Miller & Charles, roughly).
  - Two words have similar word neighbors in the corpus.

- Elements of this are from Leibniz, Harris, Firth, and Miller.
- Strictly speaking, similarity of neighbors is neither necessary nor sufficient for semantic similarity.
- But perhaps this is good enough.

Adapted slide from Hinrich Schütze
Learning word embeddings

Count-based methods:

- Compute cooccurrence statistics
- Learn high-dimensional representation
- Map sparse high-dimensional vectors to small dense representation
Word cooccurrence in Wikipedia

- corpus = English Wikipedia
- cooccurrence defined as occurrence within $k = 10$ words of each other
  - $\text{cooc.}(\text{rich}, \text{silver}) = 186$
  - $\text{cooc.}(\text{poor}, \text{silver}) = 34$
  - $\text{cooc.}(\text{rich}, \text{disease}) = 17$
  - $\text{cooc.}(\text{poor}, \text{disease}) = 162$
  - $\text{cooc.}(\text{rich}, \text{society}) = 143$
  - $\text{cooc.}(\text{poor}, \text{society}) = 228$

Adapted slide from Hinrich Schütze
Coocurrence-based Word Space

\[ \text{cooc.}(\text{poor},\text{silver}) = 34, \text{cooc.}(\text{rich},\text{silver}) = 186 \]
Cooccurrence-based Word Space

\[ \text{cooc.}(\text{poor}, \text{disease}) = 162, \text{cooc.}(\text{rich}, \text{disease}) = 17. \]
Exercise

\[ \text{ccooc.(poor,society)} = 228, \text{ccooc.(rich,society)} = 143 \]

How is it represented?
Coocurrence-based Word Space

```
coc.(poor,society)=228, cocc.(rich,society)=143
```
Dimensionality of word space

- Up to now we’ve only used two dimension words: rich and poor.
- Do this for all possible words in a corpus → high-dimensional space
- Formally, there is no difference to a two-dimensional space with three vectors.
- Note: a word can have a dual role in word space.
  - Each word can, in principle, be a dimension word, an axis of the space.
  - But each word is also a vector in that space.

Adapted slide from Hinrich Schütze
Semantic similarity

Similarity is the cosine of the angle between two word vectors.
Learning word embeddings

Count-based methods:

- Compute cooccurrence statistics
- Learn high-dimensional representation
- Map sparse high-dimensional vectors to small dense representation

Neural networks:

- Predict a word from its neighbors
- Learn (small) embedding vectors
LM Task: Given \( k \) previous words, predict the current word

\[
P(w_t|w_{t-1}, w_{t-2}, \ldots, w_{t-n})
\]

→ Learn embeddings \( C \) of words

Word embeddings learning task: Given \( k \) context words, predict the current word

→ Learn embeddings \( C \) of words
Given words $w_{t-2}$, $w_{t-1}$, $w_{t+1}$ and $w_{t+2}$, predict $w_t$ ("CBOW")

Note: Bias terms omitted for simplicity
We want the context vectors → embed words in shared space
Note: Bias terms omitted for simplicity
Getting the Word Embeddings

Note: Bias terms omitted for simplicity
Simplifications

- Remove hidden layer
- Sum over all projections
Simplifications

Remove hidden layer and sum over context
Note: Bias terms omitted for simplicity
Simplifications

- Single **logistic unit** instead of output layer
  → No need for distribution over words (only vector representation)
  → Task as binary classification problem:
    - Given input and weight matrix say if $w_t$ is current word
    - We know the correct $w_t$, how do we get the wrong ones?
      → negative sampling
Word2Vec

- BOW model (Mikolov. 2013)
- Skip-gram model:
  - Input is $w_t$
  - Prediction is $w_{t+2}, w_{t+1}, w_{t-1}$ and $w_{t-2}$
Applications

Semantic similarity:

- How similar are the words:
  - coast and shore; rich and money; happiness and disease; close and open
- WordSim-353 (Finkelstein et al. 2002)
  - Measure associations
- SimLex-999
  - Only measure semantic similarity

Other tasks:

- Use word embeddings as input features for other tasks (e.g. sentiment analysis, language modeling, named entity recognition)
Recap

Cannot fit data with **non-linear** decision boundary with linear models

**Solution**: compose non-linear functions with **neural networks**

→ Successful in many NLP applications:
  - Language modeling
  - Learning word embeddings

Feeding word embeddings into neural networks has proven successful in many NLP tasks, e.g.:
  - Sentiment Analysis
  - Named Entity Recognition
Questions?
Some Further Issues

- The backup slides (at the end) show the details of backpropagation, it is a good idea to look at these.
- Neural networks can be shown to approximate any function arbitrarily well. See the intuitive discussion of this property in this online book:
- I also highly recommend the other chapters in this book!
Thank you for your attention.
Backpropagation - Details

- The next slide shows the actual computation of backpropagation, showing the derivatives that are computed.
- The actual updates are also shown, these are more intuitive than the derivatives for many people.
Backpropagation

Compute **derivatives** in each hidden layer from layer above:

Compute derivative of error with respect to logit (output)

\[
\frac{\partial E}{\partial Z_i} = \frac{\partial E}{\partial O_i} \frac{\partial O_i}{\partial Z_i} = \frac{\partial E}{\partial O_i} O_i(1 - O_i) \quad \text{(Note: } O_i = \frac{1}{1 + e^{-Z_i}})\]

Compute derivative of error with respect to previous hidden unit

\[
\frac{\partial E}{\partial A_j} = \sum_i \frac{\partial Z_i}{\partial A_j} \frac{\partial E}{\partial Z_i} = \sum_i w_{ji} \frac{\partial E}{\partial Z_i}
\]

Compute derivative with respect to weights

\[
\frac{\partial E}{\partial w_{ji}} = \frac{\partial Z_i}{\partial w_{ji}} \frac{\partial E}{\partial Z_i} = O_i \frac{\partial E}{\partial Z_i}
\]

→ Use **recursion** to do this for every layer
Backpropagation

\[ \frac{\partial E}{\partial w_{ij}} \]

\[ Z_1 \]

\[ \ldots \]

\[ Z_K \]

\[ E(h(X), y^i) \]

\[ \text{input} \]

\[ U \]

\[ \frac{\partial E}{\partial A_j} \]

\[ V \]

\[ \frac{\partial E}{\partial O_i} \]
Weight training

**Training:** Find weight matrices $U$ and $V$ such that $h(X)$ is the **correct answer** as many times as possible.

→ Computation of $h(X)$ with **forward propagation**
→ $U$ and $V$ with error **back propagation**

For each batch of training examples

1. **Forward propagation to get predictions**
2. **Backpropagation of error**
   - Gives gradient of $E$ given **input**
3. **Modify weights (gradient descent)**
4. **Goto 1 until convergence**