Introduction to Information Retrieval

http://informationretrieval.org

IIR 6: Scoring, Term Weighting, The Vector Space Model

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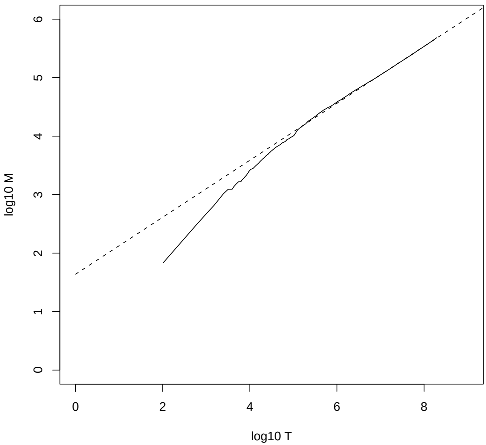
Overview

Outline

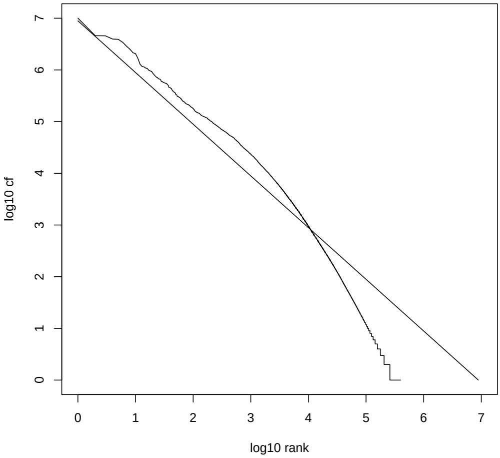
1. Recap
2. Why ranked retrieval?
3. Term frequency
4. tf-idf weighting
5. The vector space model

Heaps' law



Vocabulary size M as a function of collection size T (number of tokens) for Reuters-RCV1. For these data, the dashed line $\log_{10} M = 0.49 * \log_{10} T + 1.64$ is the best least squares fit. Thus, $M = 10^{1.64} T^{0.49}$ and $k = 10^{1.64} \approx 44$ and $b = 0.49$.

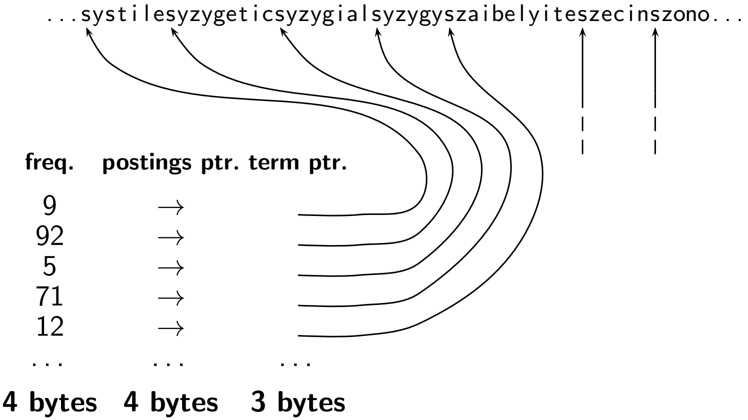
Zipf's law



$\mathrm{cf}_i \propto \frac{1}{i}$

The most frequent term (*the*) occurs cf_1 times, the second most frequent term (*of*) occurs $\mathrm{cf}_2 = \frac{1}{2}\mathrm{cf}_1$ times, the third most frequent term (*and*) occurs $\mathrm{cf}_3 = \frac{1}{3}\mathrm{cf}_1$ times etc.

Dictionary as a string



Gap encoding

	encoding	postings list									
THE	docIDs	...		283042		283043		283044		283045	...
	gaps				1		1		1		...
COMPUTER	docIDs	...		283047		283154		283159		283202	...
	gaps				107		5		43		...
ARACHNOCENTRIC	docIDs	252000		500100							
	gaps	252000	248100								

Variable byte (VB) code

- Dedicate 1 bit (high bit) to be a continuation bit c.
- If the gap G fits within 7 bits, binary-encode it in the 7 available bits and set $c = 1$.
- Else: set $c = 0$, encode high-order 7 bits and then use one or more additional bytes to encode the lower order bits using the same algorithm.

Gamma codes for gap encoding

- Represent a gap G as a pair of length and offset.
- Offset is the gap in binary, with the leading bit chopped off.
- Length is the length of offset.
- Encode length in unary code
- The Gamma code is the concatenation of length and offset.

Compression of Reuters

data structure	size in MB
dictionary, fixed-width	11.2
dictionary, term pointers into string	7.6
$\sim$, with blocking, $k = 4$	7.1
$\sim$, with blocking & front coding	5.9
collection (text, xml markup etc)	3600.0
collection (text)	960.0
T/D incidence matrix	40,000.0
postings, uncompressed (32-bit words)	400.0
postings, uncompressed (20 bits)	250.0
postings, variable byte encoded	116.0
postings, γ encoded	101.0

More on compression for information retrieval

- Zobel, Moffat: Inverted files for text search engines. ACM Computing Surveys, 2006. (a lot more detail)
- Witten, Moffat, Bell: Managing Gigabytes, 1999. (the classic)
- Büttcher, Clarke, Cormack: Information retrieval: Implementing and evaluating search engines, 2010.

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- Term frequency: This is a key ingredient for ranking.
- Tf-idf ranking: best known traditional ranking scheme
- Vector space model: Important formal model for information retrieval (along with Boolean and probabilistic models)

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- Most users don't want to wade through 1000s of results.
- This is particularly true of web search.

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- In Boolean retrieval, it takes a lot of skill to come up with a query that produces a manageable number of hits.

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- With ranking, large result sets are not an issue.
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- Premise: the ranking algorithm works: More relevant results are ranked higher than less relevant results.

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- We will look at a number of alternatives for doing this.

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- A and B don't have to be the same size.
- Always assigns a number between 0 and 1.

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  - $\text{JACCARD}(q, d) = 1/6$

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- ... instead of $|A \cap B|/|A \cup B|$ (Jaccard) for length normalization.

Outline

Binary incidence matrix

	Anthony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth	...
ANTHONY	1	1	0	0	0	1	
BRUTUS	1	1	0	1	0	0	
CAESAR	1	1	0	1	1	1	
CALPURNIA	0	1	0	0	0	0	
CLEOPATRA	1	0	0	0	0	0	
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WORSER	1	0	1	1	1	0	
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Each document is represented as a binary vector $\in \{0, 1\}^{|V|}$.

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	Anthony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth	...
ANTHONY	157	73	0	0	0	1	
BRUTUS	4	157	0	2	0	0	
CAESAR	232	227	0	2	1	0	
CALPURNIA	0	10	0	0	0	0	
CLEOPATRA	57	0	0	0	0	0	
MERCY	2	0	3	8	5	8	
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- For now: bag of words model

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- The score is 0 if none of the query terms is present in the document.

Exercise

- Compute the Jaccard matching score and the tf matching score for the following query-document pairs.
- q: [information on cars] d: "all you've ever wanted to know about cars"
- q: [information on cars] d: "information on trucks, information on planes, information on trains"
- q: [red cars and red trucks] d: "cops stop red cars more often"

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- In addition, to term frequency (the frequency of the term in the document) . . .
- . . . we also want to use the frequency of the term in the collection for weighting and ranking.

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- → For frequent terms like GOOD, INCREASE, and LINE, we want positive weights ...
- ... but lower weights than for rare terms.

Document frequency

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- We want low (positive) weights for frequent words like GOOD, INCREASE, and LINE.

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- We want low (positive) weights for frequent words like GOOD, INCREASE, and LINE.
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- The document frequency is the number of documents in the collection that the term occurs in.

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- Note that we use the log transformation for both term frequency and document frequency.

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animal	100	
sunday	1000	
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animal	100	4
sunday	1000	3
fly	10,000	2
under	100,000	1
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- For example, in the query "arachnocentric line", idf weighting increases the relative weight of ARACHNOCENTRIC and decreases the relative weight of LINE.
- idf has little effect on ranking for one-term queries.

Collection frequency vs. Document frequency

word	collection frequency	document frequency
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TRY	10422	8760

- Collection frequency of t: number of tokens of t in the collection
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- Which word is a better search term (and should get a higher weight)?
- This example suggests that df (and idf) is better for weighting than cf (and "icf").

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- Best known weighting scheme in information retrieval
- Alternative names: tf.idf, tf x idf

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  - ...increases with the number of occurrences within a document. (term frequency)
  - ...increases with the rarity of the term in the collection. (inverse document frequency)

Exercise: Term, collection and document frequency

Quantity	Symbol	Definition
term frequency	$\text{tf}_{t,d}$	number of occurrences of t in d
document frequency	df_t	number of documents in the collection that t occurs in
collection frequency	cf_t	total number of occurrences of t in the collection

- Relationship between df and cf?
- Relationship between tf and cf?
- Relationship between tf and df?

Outline

1. Recap
2. Why ranked retrieval?
3. Term frequency
4. tf-idf weighting
5. The vector space model

Binary incidence matrix

	Anthony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth	...
ANTHONY	1	1	0	0	0	1	
BRUTUS	1	1	0	1	0	0	
CAESAR	1	1	0	1	1	1	
CALPURNIA	0	1	0	0	0	0	
CLEOPATRA	1	0	0	0	0	0	
MERCY	1	0	1	1	1	1	
WORSER	1	0	1	1	1	0	
...							

Each document is represented as a binary vector $\in \{0,1\}^{|V|}$.

Count matrix

	Anthony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth	...
ANTHONY	157	73	0	0	0	1	
BRUTUS	4	157	0	2	0	0	
CAESAR	232	227	0	2	1	0	
CALPURNIA	0	10	0	0	0	0	
CLEOPATRA	57	0	0	0	0	0	
MERCY	2	0	3	8	5	8	
WORSER	2	0	1	1	1	5	
...							

Each document is now represented as a count vector $\in \mathbb{N}^{|V|}$.

Binary → count → weight matrix

	Anthony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth	...
Anthony	5.25	3.18	0.0	0.0	0.0	0.35	
Brutus	1.21	6.10	0.0	1.0	0.0	0.0	
Caesar	8.59	2.54	0.0	1.51	0.25	0.0	
Calpurnia	0.0	1.54	0.0	0.0	0.0	0.0	
Cleopatra	2.85	0.0	0.0	0.0	0.0	0.0	
mercy	1.51	0.0	1.90	0.12	5.25	0.88	
worser	1.37	0.0	0.11	4.15	0.25	1.95	
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Each document is now represented as a real-valued vector of tf-idf weights $\in \mathbb{R}^{|V|}$.

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- Very high-dimensional: tens of millions of dimensions when you apply this to web search engines
- Each vector is very sparse - most entries are zero.

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- Key idea 1: do the same for queries: represent them as vectors in the high-dimensional space
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- proximity = similarity
- proximity ≈ negative distance
- Recall: We're doing this because we want to get away from the you're-either-in-or-out, feast-or-famine Boolean model.
- Instead: rank relevant documents higher than nonrelevant documents

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- Euclidean distance is a bad idea . . .
- . . . because Euclidean distance is large for vectors of different lengths.

Why distance is a bad idea

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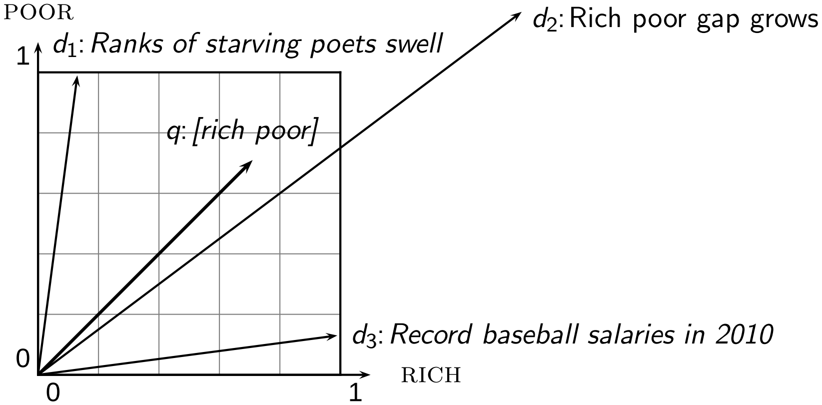

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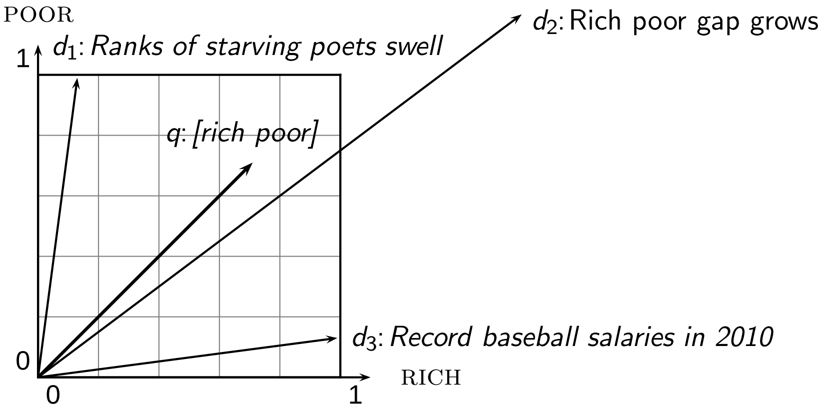

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Questions about basic vector space setup?

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- Thought experiment: take a document d and append it to itself. Call this document d'. d' is twice as long as d.
- "Semantically" d and d' have the same content.
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- . . . even though the Euclidean distance between the two documents can be quite large.

From angles to cosines

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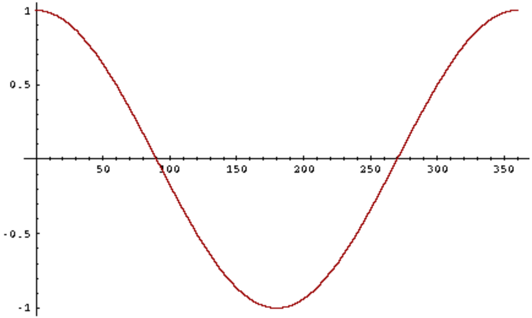
- Cosine is a monotonically decreasing function of the angle for the interval $[0^\circ, 180^\circ]$

Cosine

Cosine



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- . . . since after normalization: $||x||_2 = \sqrt{\sum_i x_i^2} = 1.0$
- As a result, longer documents and shorter documents have weights of the same order of magnitude.
- Effect on the two documents d and d' (d appended to itself) from earlier slide: they have identical vectors after length-normalization.

Cosine similarity between query and document

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$$\cos(\vec{q},\vec{d}) = \textsc{sim}(\vec{q},\vec{d}) = \frac{\vec{q}\cdot\vec{d}}{|\vec{q}||\vec{d}|} = \frac{\sum_{i=1}^{|V|} q_i d_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2}\sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

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- This is the cosine similarity of $\vec{q}$ and $\vec{d}$ . . . . . . or, equivalently, the cosine of the angle between $\vec{q}$ and $\vec{d}$.

Cosine for normalized vectors

- For normalized vectors, the cosine is equivalent to the dot product or scalar product.
- $\cos(\vec{q}, \vec{d}) = \vec{q} \cdot \vec{d} = \sum_i q_i \cdot d_i$
  - (if $\vec{q}$ and $\vec{d}$ are length-normalized).

Cosine similarity illustrated

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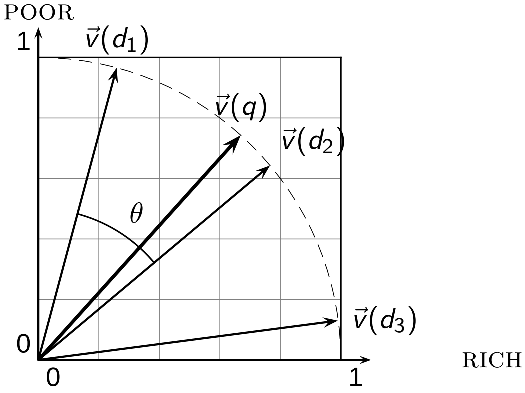

Cosine: Example

How similar are these novels?

SaS: Sense and Sensibility

PaP: Pride and Prejudice

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term frequencies (counts)

term	SaS	PaP	WH
AFFECTION	115	58	20
JEALOUS	10	7	11
GOSSIP	2	0	6
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(To simplify this example, we don't do idf weighting.)

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log frequency weighting & cosine normalization

term	SaS	PaP	WH
AFFECTION	0.789	0.832	0.524
JEALOUS	0.515	0.555	0.465
GOSSIP	0.335	0.0	0.405
WUTHERING	0.0	0.0	0.588

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- cos(SaS,PaP) $\approx$ $0.789 * 0.832 + 0.515 * 0.555 + 0.335 * 0.0 + 0.0 * 0.0 \approx 0.94$.

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AFFECTION	0.789	0.832	0.524
JEALOUS	0.515	0.555	0.465
GOSSIP	0.335	0.0	0.405
WUTHERING	0.0	0.0	0.588

- $\cos(\text{SaS},\text{PaP}) \approx$ $0.789 * 0.832 + 0.515 * 0.555 + 0.335 * 0.0 + 0.0 * 0.0 \approx 0.94$.
- $\cos(\text{SaS},\text{WH}) \approx 0.79$

Cosine: Example

log frequency weighting

term	SaS	PaP	WH
AFFECTION	3.06	2.76	2.30
JEALOUS	2.0	1.85	2.04
GOSSIP	1.30	0	1.78
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log frequency weighting & cosine normalization

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- cos(SaS,PaP) $\approx$ $0.789 * 0.832 + 0.515 * 0.555 + 0.335 * 0.0 + 0.0 * 0.0 \approx 0.94$.
- cos(SaS,WH) ≈ 0.79
- cos(PaP,WH) ≈ 0.69
- Why do we have cos(SaS,PaP) $>$ cos(SAS,WH)?

Computing the cosine score

Computing the cosine score

CosineScore(q)

float $Scores[N] = 0$
float $Length[N]$
for each query term t
do calculate $w_{t,q}$ and fetch postings list for t
    for each pair(d, $tf_{t,d}$) in postings list
    do $Scores[d] += w_{t,d} \times w_{t,q}$
Read the array *Length*
for each d
do $Scores[d] = Scores[d]/Length[d]$
return Top K components of $Scores[]$

Components of tf-idf weighting

Term frequency		Document frequency		Normalization	
n (natural)	$\mathrm{tf}_{t,d}$	n (no)	1	n (none)	1
l (logarithm)	$1 + \log(\mathrm{tf}_{t,d})$	t (idf)	$\log \frac{N}{\mathrm{df}_t}$	c (cosine)	$\frac{1}{\sqrt{w_1^2 + w_2^2 + \ldots + w_M^2}}$
a (augmented)	$0.5 + \frac{0.5 \times \mathrm{tf}_{t,d}}{\max_t(\mathrm{tf}_{t,d})}$	p (prob idf)	$\max\{0, \log \frac{N - \mathrm{df}_t}{\mathrm{df}_t}\}$	u (pivoted unique)	$1/u$
b (boolean)	$\begin{cases} 1 & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0 & \text{otherwise} \end{cases}$			b (byte size)	$1/\mathit{CharLength}^{\alpha}$, $\alpha < 1$
L (log ave)	$\frac{1 + \log(\mathrm{tf}_{t,d})}{1 + \log(\mathrm{ave}_{t \in d}(\mathrm{tf}_{t,d}))}$				

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Best known combination of weighting options

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Default: no weighting

tf-idf example

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- Example: lnc.ltn
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- Isn't it bad to not idf-weight the document?

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- Example: lnc.ltn
- document: logarithmic tf, no df weighting, cosine normalization
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- Example query: “best car insurance”

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- document: logarithmic tf, no df weighting, cosine normalization
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- Isn't it bad to not idf-weight the document?
- Example query: "best car insurance"
- Example document: "car insurance auto insurance"

tf-idf example: lnc.ltn

Query: "best car insurance". Document: "car insurance auto insurance".

word	query					document				product
	tf-raw	tf-wght	df	idf	weight	tf-raw	tf-wght	weight	n'lized	
auto										
best										
car										
insurance										

Key to columns: tf-raw: raw (unweighted) term frequency, tf-wght: logarithmically weighted term frequency, df: document frequency, idf: inverse document frequency, weight: the final weight of the term in the query or document, n'lized: document weights after cosine normalization, product: the product of final query weight and final document weight

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word	query					document				product
	tf-raw	tf-wght	df	idf	weight	tf-raw	tf-wght	weight	n'lized	
auto	0									
best	1									
car	1									
insurance	1									

Key to columns: tf-raw: raw (unweighted) term frequency, tf-wght: logarithmically weighted term frequency, df: document frequency, idf: inverse document frequency, weight: the final weight of the term in the query or document, n'lized: document weights after cosine normalization, product: the product of final query weight and final document weight

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auto	0					1				
best	1					0				
car	1					1				
insurance	1					2				

Key to columns: tf-raw: raw (unweighted) term frequency, tf-wght: logarithmically weighted term frequency, df: document frequency, idf: inverse document frequency, weight: the final weight of the term in the query or document, n'lized: document weights after cosine normalization, product: the product of final query weight and final document weight

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best	1	1				0	0			
car	1	1				1	1			
insurance	1	1				2	1.3			

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best	1	1	50000			0	0			
car	1	1	10000			1	1			
insurance	1	1	1000			2	1.3			

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insurance	1	1	1000	3.0	3.0	2	1.3	1.3	0.68	

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$\sqrt{1^2 + 0^2 + 1^2 + 1.3^2} \approx 1.92$
$1/1.92 \approx 0.52$
$1.3/1.92 \approx 0.68$

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car	1	1	10000	2.0	2.0	1	1	1	0.52	1.04
insurance	1	1	1000	3.0	3.0	2	1.3	1.3	0.68	2.04

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Final similarity score between query and document: $\sum_i w_{qi} \cdot w_{di} = 0 + 0 + 1.04 + 2.04 = 3.08$

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Questions?

Summary: Ranked retrieval in the vector space model

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- Represent the query as a weighted tf-idf vector

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- Compute the cosine similarity between the query vector and each document vector
- Rank documents with respect to the query

Summary: Ranked retrieval in the vector space model

- Represent the query as a weighted tf-idf vector
- Represent each document as a weighted tf-idf vector
- Compute the cosine similarity between the query vector and each document vector
- Rank documents with respect to the query
- Return the top K (e.g., $K = 10$) to the user

Take-away today

- Ranking search results: why it is important (as opposed to just presenting a set of unordered Boolean results)
- Term frequency: This is a key ingredient for ranking.
- Tf-idf ranking: best known traditional ranking scheme
- Vector space model: Important formal model for information retrieval (along with Boolean and probabilistic models)

Resources

- Chapters 6 and 7 of IIR
- Resources at `http://cislmu.org`
  - Vector space for dummies
  - Exploring the similarity space (Moffat and Zobel, 2005)
  - Okapi BM25 (a state-of-the-art weighting method, 11.4.3 of IIR)