Einführung in die Computerlinguistik
Decision Trees

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2015-11-23

This lecture is based on
Russell and Norvig's introduction
to artificial intelligence.

Take-away today

- Introduction to decision trees
- (Almost) fully understand one complex machine learning model
- Basis for hands-on training and applying this model
  in next practical exercise
- Exam: questions testing basic understanding of decision trees,
  but no formulas

Overview

1. Decision trees
2. NLTK

Outline

1. Decision trees
2. NLTK

Attributes

As an example, we will build a decision tree to decide whether to wait for a table at a restaurant. The aim here is to learn a definition for the **goal predicate** *WillWait*. First we list the attributes that we will consider as part of the input:

1. *Alternate*: whether there is a suitable alternative restaurant nearby.
2. *Bar*: whether the restaurant has a comfortable bar area to wait in.
3. *Fri/Sat*: true on Fridays and Saturdays.
4. *Hungry*: whether we are hungry.
5. *Patrons*: how many people are in the restaurant (values are *None*, *Some*, and *Full*).
6. *Price*: the restaurant's price range ($, $$, $$$).
7. *Raining*: whether it is raining outside.
8. *Reservation*: whether we made a reservation.
9. *Type*: the kind of restaurant (French, Italian, Thai, or burger).
10. *WaitEstimate*: the wait estimated by the host (0–10 minutes, 10–30, 30–60, or >60).

Decision tree for deciding whether to wait

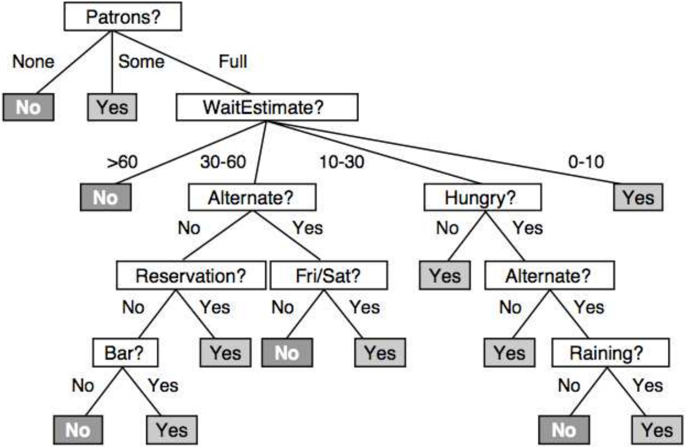

Figure 18.2 A decision tree for deciding whether to wait for a table. (explain one path)

Expressiveness of decision trees

18.3.2 Expressiveness of decision trees

A Boolean decision tree is equivalent to a logical expression of the form

$$Goal \Leftrightarrow (Path_1 \vee Path_2 \vee \cdots),$$

where each $Path_i$ has the form

$$Path = (A_i = a_i \wedge A_j = a_j \wedge \cdots),$$

that is, the goal is true if and only if there is a path through the tree that ends in a positive result. Since this is equivalent to disjunctive normal form, that means that any function in propositional logic can be expressed as a decision tree. As an example, the rightmost path in Figure 18.2 is

$$Path = (Patrons = Full \wedge WaitEstimate = 0\text{–}10).$$

Training set

Example	Input Attributes										Goal
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
$\mathbf{x}_1$	*Yes*	*No*	*No*	*Yes*	*Some*	*$$$*	*No*	*Yes*	*French*	*0–10*	$y_1 =$ *Yes*
$\mathbf{x}_2$	*Yes*	*No*	*No*	*Yes*	*Full*	*$*	*No*	*No*	*Thai*	*30–60*	$y_2 =$ *No*
$\mathbf{x}_3$	*No*	*Yes*	*No*	*No*	*Some*	*$*	*No*	*No*	*Burger*	*0–10*	$y_3 =$ *Yes*
$\mathbf{x}_4$	*Yes*	*No*	*Yes*	*Yes*	*Full*	*$*	*Yes*	*No*	*Thai*	*10–30*	$y_4 =$ *Yes*
$\mathbf{x}_5$	*Yes*	*No*	*Yes*	*No*	*Full*	*$$$*	*No*	*Yes*	*French*	*>60*	$y_5 =$ *No*
$\mathbf{x}_6$	*No*	*Yes*	*No*	*Yes*	*Some*	*$$*	*Yes*	*Yes*	*Italian*	*0–10*	$y_6 =$ *Yes*
$\mathbf{x}_7$	*No*	*Yes*	*No*	*No*	*None*	*$*	*Yes*	*No*	*Burger*	*0–10*	$y_7 =$ *No*
$\mathbf{x}_8$	*No*	*No*	*No*	*Yes*	*Some*	*$$*	*Yes*	*Yes*	*Thai*	*0–10*	$y_8 =$ *Yes*
$\mathbf{x}_9$	*No*	*Yes*	*Yes*	*No*	*Full*	*$*	*Yes*	*No*	*Burger*	*>60*	$y_9 =$ *No*
$\mathbf{x}_{10}$	*Yes*	*Yes*	*Yes*	*Yes*	*Full*	*$$$*	*No*	*Yes*	*Italian*	*10–30*	$y_{10} =$ *No*
$\mathbf{x}_{11}$	*No*	*No*	*No*	*No*	*None*	*$*	*No*	*No*	*Thai*	*0–10*	$y_{11} =$ *No*
$\mathbf{x}_{12}$	*Yes*	*Yes*	*Yes*	*Yes*	*Full*	*$*	*No*	*No*	*Burger*	*30–60*	$y_{12} =$ *Yes*

Usefulness of attributes

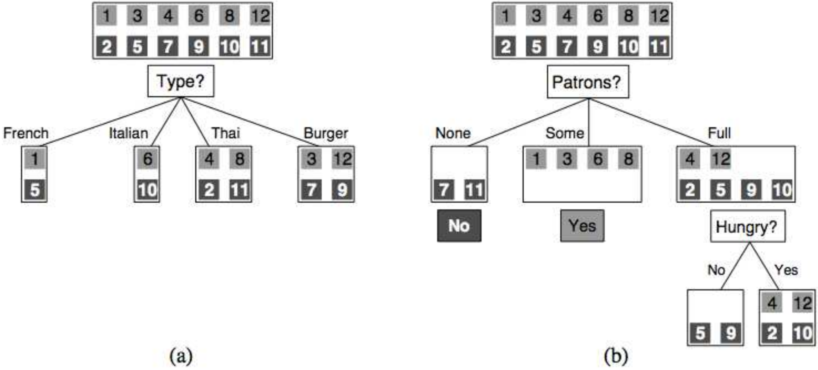

Figure 18.4 Splitting the examples by testing on attributes. At each node we show the positive (light boxes) and negative (dark boxes) examples remaining. (a) Splitting on *Type* brings us no nearer to distinguishing between positive and negative examples. (b) Splitting on *Patrons* does a good job of separating positive and negative examples. After splitting on *Patrons*, *Hungry* is a fairly good second test.

Decision tree learning : Importance?

```
function DECISION-TREE-LEARNING(examples, attributes, parent_examples) returns
a tree

  if examples is empty then return PLURALITY-VALUE(parent_examples)
  else if all examples have the same classification then return the classification
  else if attributes is empty then return PLURALITY-VALUE(examples)
  else
      attr ← argmax_{a ∈ attributes} IMPORTANCE(a, examples)
      tree ← a new decision tree with root test attr
      for each value v_i of attr do
          exs ← {e : e ∈ examples and e.attr = v_i}
          subtree ← DECISION-TREE-LEARNING(exs, attributes − attr, examples)
          add a branch to tree with label (attr = v_i) and subtree subtree
      return tree
```

Figure 18.5 The decision-tree learning algorithm. The function IMPORTANCE is described in Section 18.3.4. The function PLURALITY-VALUE selects the most common output value among a set of examples, breaking ties randomly.

Induced decision tree

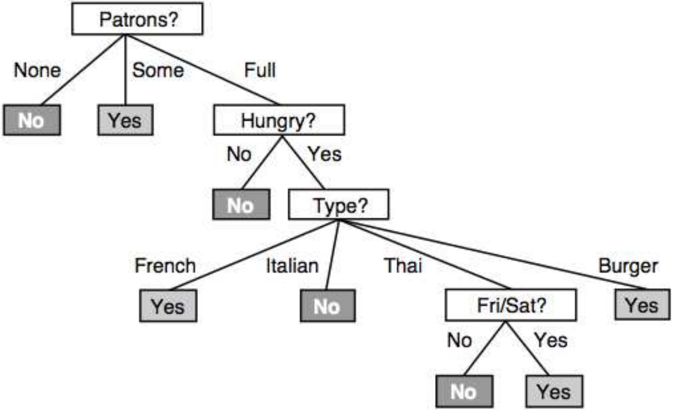

Figure 18.6 The decision tree induced from the 12-example training set.

Hand-designed vs. induced trees

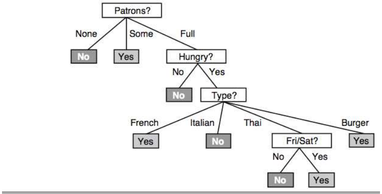

Figure 18.6 The decision tree induced from the 12-example training set.

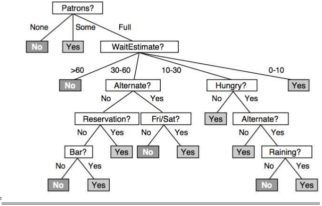

Figure 18.2 A decision tree for deciding whether to wait for a table.

Decision tree learning : Importance?

```
function DECISION-TREE-LEARNING(examples, attributes, parent_examples) returns
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Entropy

$$
\begin{aligned}
H(V) &= \sum_i P(v_i) \log_2 \frac{1}{P(v_i)} \\
&= -\sum_i P(v_i) \log_2 P(v_i)
\end{aligned}
$$

Entropy: Restaurant example

$$
\begin{aligned}
H(V) &= \sum_i P(v_i)\log_2 \frac{1}{P(v_i)} \\
&= -\sum_i P(v_i)\log_2 P(v_i)
\end{aligned}
$$

$$
H(\text{Goal}) = -\left(\frac{p}{p+n}\log_2 \frac{p}{p+n} + \frac{n}{p+n}\log_2 \frac{n}{p+n}\right)
$$

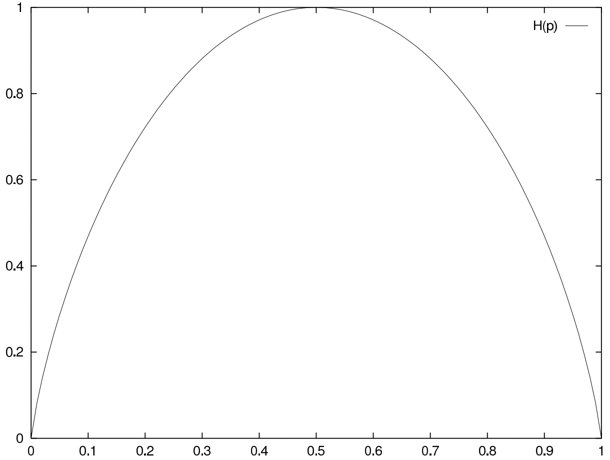
p is number of positive examples (“will wait”),

n is number of negative examples (“will not wait”)

Entropy: Restaurant example

$$
\begin{aligned}
H(\text{Goal}) &= -(\frac{p}{p+n}\log_2\frac{p}{p+n} + \frac{n}{p+n}\log_2\frac{n}{p+n}) \\
&= -(\frac{6}{6+6}\log_2\frac{6}{6+6} + \frac{6}{6+6}\log_2\frac{6}{6+6}) \\
&= -2\frac{6}{6+6}\log_2\frac{6}{6+6} \\
&= \log_2 2 \\
&= 1
\end{aligned}
$$

Plot of entropy



Notation

$$H(T(q)) = -q\log_2 q - (1-q)\log_2(1-q)$$

Remainder = Remaining uncertainty

$$\text{Remainder}(A) = \sum_{i=1}^{|v(A)|} \frac{p_i + n_i}{p + n} H(T(\frac{p_i}{p_i + n_i}))$$

p_i is the number of positive examples that have attribute value v_i ($A = v_i$) and n_i is the number of negative examples that have attribute value v_i ($A = v_i$).

Information gain

$$\text{Gain}(A) = H(T(\frac{p}{p+n})) - \text{Remainder}(A)$$

Information gain for Pt = Patrons and Tp = Type

$$
\begin{aligned}
\text{Gain(Tp)} &= 1 - [\frac{2}{12}H(T(\frac{1}{2})) + \frac{2}{12}H(T(\frac{1}{2})) + \frac{4}{12}H(T(\frac{2}{4})) + \frac{4}{12}H(T(\frac{2}{4}))] \\
&= 1 - [\frac{2}{12} + \frac{2}{12} + \frac{4}{12} + \frac{4}{12}] \\
&= 0 \text{ bits} \\
\text{Gain(Pt)} &= 1 - [\frac{2}{12}H(T(\frac{0}{2})) + \frac{4}{12}H(T(\frac{4}{4})) + \frac{6}{12}H(T(\frac{2}{6}))] \\
&= 1 - [0 + 0 + \frac{1}{2}H(T(\frac{1}{3}))] \\
&\approx 0.541 \text{ bits}
\end{aligned}
$$

Entropy exercise (goal)

example	decision	type	day of week	colleague?
x_1	yes	french	saturday	"let's stay"
x_2	no	thai	friday	"let's go"
x_3	yes	burger	saturday	"let's stay"
x_4	yes	thai	sunday	"let's stay"
x_5	no	french	friday	"let's go"
x_6	yes	italian	sunday	"let's stay"
x_7	no	burger	friday	"let's go"
x_8	yes	thai	sunday	"let's stay"
x_9	no	burger	friday	"not sure"
x_{10}	no	italian	friday	"not sure"
x_{11}	no	thai	friday	"not sure"
x_{12}	yes	burger	sunday	"not sure"

Decision tree learning : Importance?

function DECISION-TREE-LEARNING(*examples*, *attributes*, *parent_examples*) **returns** a tree

```
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```

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Outline

NLTK decision tree demo

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