Einführung in die Computerlinguistik
Text Classification and Naive Bayes

Hinrich Schütze

Center for Information and Language Processing

2019-01-07

Outline

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1. Text classification
2. Naive Bayes
3. NB theory
4. Evaluation of TC

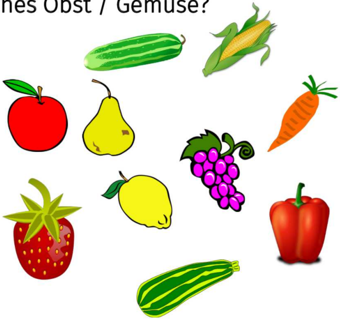
A text classification task: Email spam filtering

```
From: ‘‘’’ <takworlld@hotmail.com>
Subject: real estate is the only way... gem  oalvgkay

Anyone can buy real estate with no money down

Stop paying rent TODAY !

There is no need to spend hundreds or even thousands for similar courses

I am 22 years old and I have already purchased 6 properties using the
methods outlined in this truly INCREDIBLE ebook.

Change your life NOW !

=================================================
Click Below to order:
http://www.wholesaledaily.com/sales/nmd.htm
=================================================
```

Mustererkennung (pattern recognition)

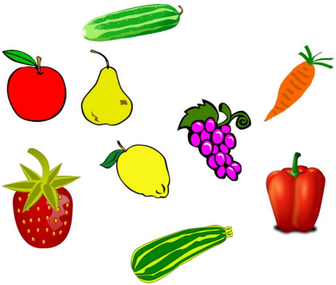
Kamera

Welches Obst / Gemüse?



Was sind mögliche Erkennungsmerkmale?

Mustererkennung (pattern recognition)



Merkmale/Attribute:
- Farbe
- Größe
- Form
- ...

Beispiele:
Attribute/Werte + richtige Klasse

↓

Algorithmus
(Maschinelles Lernen)

↓

neues Objekt → Classifier → *Zitrone (80%)*
Birne (20%)

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Using a learning method or learning algorithm, we then wish to learn a classifier γ that maps documents to classes:

$$\gamma : \mathbb{X} \rightarrow \mathbb{C}$$

We can view sentences also as documents – so “document” refers to any piece of text we want to classify.

Formal definition of TC: Application/Testing

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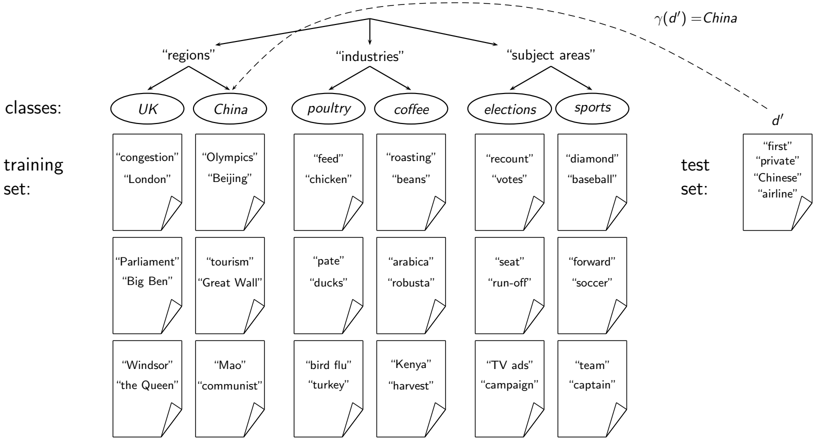
Given: a description $d \in \mathbb{X}$ of a document

Determine: $\gamma(d) \in \mathbb{C}$, that is,
determine the class that is most appropriate for d

Topic classification



Applications of text classification

- Language identification
  (classes: English vs French vs . . .)
- The automatic detection of spam pages
  (spam vs nonspam)
- Sentiment analysis:
  Is a movie or product review positive or negative
  (positive vs negative)
- Topic-specific or *vertical* search:
  Restrict search to a "vertical" like "related to health"
  (classes: relevant to vertical vs not)

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- $\rightarrow$ We need automatic methods for classification.

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- Building and maintaining rule-based classification systems is cumbersome and expensive.

A Verity topic (a complex classification rule)

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```
comment line                 # Beginning of art topic definition
top-level topic              art ACCRUE
                                 /author = "fsmith"
topic definition modifiers       /date   = "30-Dec-01"
                                 /annotation = "Topic created
                                                by fsmith"
subtopictopic                * 0.70 performing-arts ACCRUE
  evidencetopic              ** 0.50 WORD
  topicdefinition modifier       /wordtext = ballet
  evidencetopic              ** 0.50 STEM
  topicdefinition modifier       /wordtext = dance
  evidencetopic              ** 0.50 WORD
  topicdefinition modifier       /wordtext = opera
  evidencetopic              ** 0.30 WORD
  topicdefinition modifier       /wordtext = symphony
subtopic                     * 0.70 visual-arts ACCRUE
                             ** 0.50 WORD
                                 /wordtext = painting
                             ** 0.50 WORD
                                 /wordtext = sculpture
subtopic                     * 0.70 film ACCRUE
                             ** 0.50 STEM
                                 /wordtext = film
subtopic                     ** 0.50 motion-picture PHRASE
                             *** 1.00 WORD
                                 /wordtext = motion
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```

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- (i) Supervised learning of a the classification function γ and (ii) application of γ to classifying new documents
- We will look at one method for doing this: Naive Bayes
- No free lunch: requires hand-classified training data
- But this manual classification can be done by non-experts.

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- If a document's terms do not provide clear evidence for one class vs. another, we choose the c with highest $P(c)$.

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  Find the "best" class
- The best class is the most likely or maximum a posteriori (MAP) class c_{map}:

$$c_{\text{map}} = \text{argmax}_{c \in \mathbb{C}} \hat{P}(c|d) = \text{argmax}_{c \in \mathbb{C}} \ \hat{P}(c) \prod_{1 \leq k \leq n_d} \hat{P}(t_k|c)$$

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- Since $\log(xy) = \log(x) + \log(y)$, we can sum log probabilities instead of multiplying probabilities.
- Since log is a monotonic function, the class with the highest score does not change.
- So what we usually compute in practice is:

$$c_{\text{map}} = \text{argmax}_{c \in \mathbb{C}} \, [\log \hat{P}(c) + \sum_{1 \leq k \leq n_d} \log \hat{P}(t_k|c)]$$

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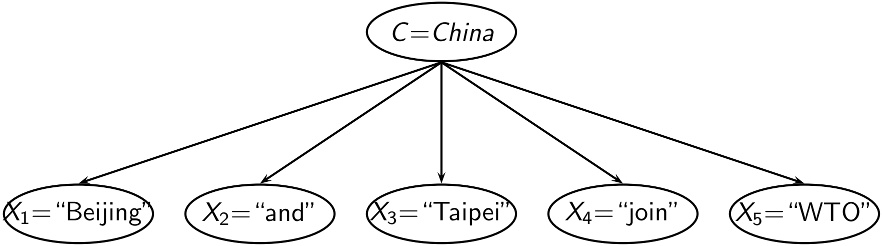

$$\begin{aligned} P(China|d) \propto\ & P(China) \cdot P(\text{“Beijing”}|China) \cdot P(\text{“and”}|China) \\ & \cdot P(\text{“Taipei”}|China) \cdot P(\text{“join”}|China) \cdot P(\text{“WTO”}|China) \end{aligned}$$

- If "WTO" never occurs in class China in the train set:

$$\hat{P}(\text{“WTO”}|China) = \frac{T_{China,\text{“WTO”}}}{\sum_{t' \in V} T_{China,t'}} = \frac{0}{\sum_{t' \in V} T_{China,t'}} = 0$$

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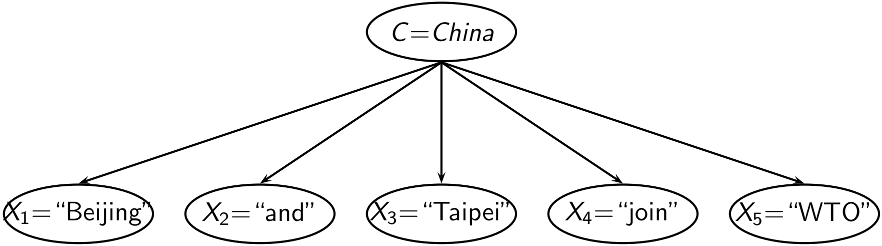

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- $\rightarrow$ We will get $P(China|d) = 0$ for any document that contains WTO!

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$$\hat{P}(t|c) = \frac{T_{ct} + 1}{\sum_{t' \in V}(T_{ct'} + 1)} = \frac{T_{ct} + 1}{(\sum_{t' \in V} T_{ct'}) + B}$$

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- For a new document, for each class, compute sum of (i) log of prior and (ii) logs of conditional probabilities of the terms
- Assign the document to the class with the largest score

Naive Bayes: Training

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TrainMultinomialNB($\mathbb{C}, \mathbb{D}$)

$V \leftarrow$ ExtractVocabulary($\mathbb{D}$)
$N \leftarrow$ CountDocs($\mathbb{D}$)
for each $c \in \mathbb{C}$
do $N_c \leftarrow$ CountDocsInClass($\mathbb{D}, c$)
    $prior[c] \leftarrow N_c / N$
    $text_c \leftarrow$ ConcatenateTextOfAllDocsInClass($\mathbb{D}, c$)
    for each $t \in V$
    do $T_{ct} \leftarrow$ CountTokensOfTerm($text_c, t$)
    for each $t \in V$
    do $condprob[t][c] \leftarrow \frac{T_{ct}+1}{\sum_{t'}(T_{ct'}+1)}$
return $V, prior, condprob$

Naive Bayes: Testing

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ApplyMultinomialNB($\mathbb{C}$, V, *prior*, *condprob*, d)

$W \leftarrow$ ExtractTokensFromDoc(V, d)
for each $c \in \mathbb{C}$
do $\mathit{score}[c] \leftarrow \log \mathit{prior}[c]$
    for each $t \in W$
    do $\mathit{score}[c] += \log \mathit{condprob}[t][c]$
return $\mathrm{argmax}_{c \in \mathbb{C}}\, \mathit{score}[c]$

Exercise: Estimate parameters, classify test set

	docID	words in document	in $c = China$?
training set	1	Chinese Beijing Chinese	yes
	2	Chinese Chinese Shanghai	yes
	3	Chinese Macao	yes
	4	Tokyo Japan Chinese	no
test set	5	Chinese Chinese Chinese Tokyo Japan	?

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$$\hat{P}(t|c) = \frac{T_{ct} + 1}{\sum_{t' \in V}(T_{ct'} + 1)} = \frac{T_{ct} + 1}{(\sum_{t' \in V} T_{ct'}) + B}$$

(B is the number of bins – in this case the number of different words or the size of the vocabulary $|V| = M$)

$$c_{\text{map}} = \text{argmax}_{c \in \mathbb{C}} \, [\hat{P}(c) \cdot \prod_{1 \leq k \leq n_d} \hat{P}(t_k|c)]$$

Example: Parameter estimates

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Priors: $\hat{P}(c) = 3/4$ and $\hat{P}(\overline{c}) = 1/4$
Conditional probabilities:

$$\begin{aligned}
\hat{P}(\text{“Chinese”}|c) &= (5+1)/(8+6) = 6/14 = 3/7 \\
\hat{P}(\text{“Tokyo”}|c) = \hat{P}(\text{“Japan”}|c) &= (0+1)/(8+6) = 1/14 \\
\hat{P}(\text{“Chinese”}|\overline{c}) &= (1+1)/(3+6) = 2/9 \\
\hat{P}(\text{“Tokyo”}|\overline{c}) = \hat{P}(\text{“Japan”}|\overline{c}) &= (1+1)/(3+6) = 2/9
\end{aligned}$$

The denominators are $(8+6)$ and $(3+6)$ because the lengths of $text_c$ and $text_{\overline{c}}$ are 8 and 3, respectively, and because the constant B is 6 as the vocabulary consists of six terms.

Example: Classification

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$$\hat{P}(c|d_5) \propto 3/4 \cdot (3/7)^3 \cdot 1/14 \cdot 1/14 \approx 0.0003$$
$$\hat{P}(\overline{c}|d_5) \propto 1/4 \cdot (2/9)^3 \cdot 2/9 \cdot 2/9 \approx 0.0001$$

Thus, the classifier assigns the test document to c = *China*. The reason for this classification decision is that the three occurrences of the positive indicator "Chinese" in d_5 outweigh the occurrences of the two negative indicators "Japan" and "Tokyo".

UNK – unknown words

UNK

An UNK is a word that occurs in the test set,
but did not occur in the training set.

- Option 1: Simply ignore UNKs
- Option 2: Add UNK to the training vocabulary
  - All counts $T_{c\text{UNK}}$ are zero (since UNK does not occur in training set).
  - All words in the test set that did not occur in the training set are replaced by "UNK".

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- Now we want to gain a better understanding of the properties of Naive Bayes.
- We will formally derive the classification rule ...
- ... and make our assumptions explicit.

Derivation of Naive Bayes rule

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We want to find the class that is most likely given the document:

$$c_{\text{map}} = \text{argmax}_{c \in \mathbb{C}} P(c|d)$$

Apply Bayes rule $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$:

$$c_{\text{map}} = \text{argmax}_{c \in \mathbb{C}} \frac{P(d|c)P(c)}{P(d)}$$

Drop denominator since $P(d)$ is the same for all classes:

$$c_{\text{map}} = \text{argmax}_{c \in \mathbb{C}} P(d|c)P(c)$$

Too many parameters / sparseness

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- We would need a very, very large number of training examples to estimate that many parameters.
- This is the problem of data sparseness.

Bag of words model

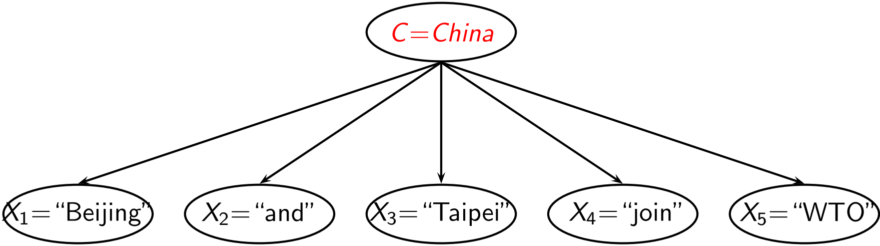
To reduce the number of parameters to a manageable size, we make the Naive Bayes conditional independence assumption:

$$P(d|c) = P(\langle t_1, \ldots, t_{n_d} \rangle | c) = \prod_{1 \leq k \leq n_d} P(X_k = t_k | c)$$

We assume that the probability of observing the conjunction of attributes is equal to the product of the individual probabilities $P(X_k = t_k|c)$.
Recall from earlier the estimates for these conditional probabilities: $\hat{P}(t|c) = \frac{T_{ct}+1}{(\sum_{t' \in V} T_{ct'})+B}$
Bag of words model

Generative model

Generative model



$P(c|d) \propto P(c) \prod_{1 \leq k \leq n_d} P(t_k|c)$

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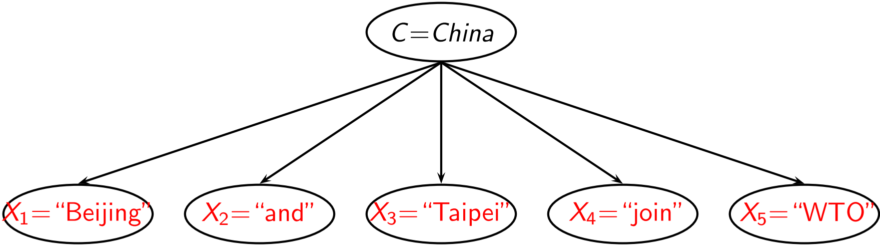
Generative model



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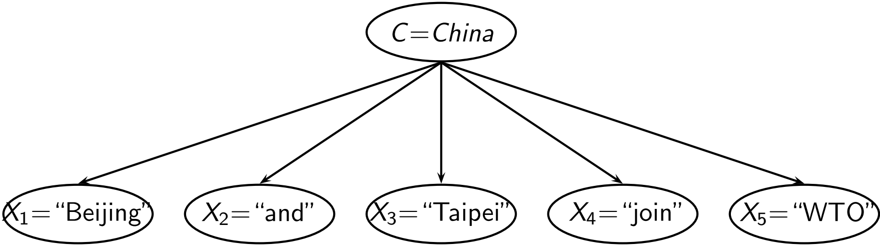

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- Generate a class with probability $P(c)$
- Generate each of the words (in their respective positions), conditional on the class, but independent of each other, with probability $P(t_k|c)$
- To classify docs, we "reengineer" this process and find the class that is most likely to have generated the doc.

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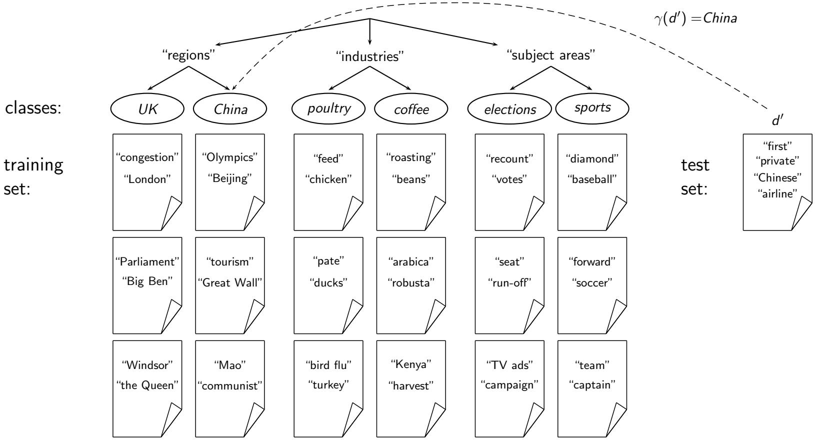
- Low storage requirements

Outline

Evaluation on Reuters



Example: The Reuters collection

symbol	statistic	value
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M	word types	400,000

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type of class	number	examples
region	366	UK, China
industry	870	poultry, coffee
subject area	126	elections, sports

A Reuters document

REUTERS

You are here: Home > News > Science > Article

Go to a Section: U.S. International Business Markets Politics Entertainment Technology Sports Oddly Enoug

Extreme conditions create rare Antarctic clouds

Tue Aug 1, 2006 3:20am ET

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[-] Text [+]

SYDNEY (Reuters) - Rare, mother-of-pearl colored clouds caused by extreme weather conditions above Antarctica are a possible indication of global warming, Australian scientists said on Tuesday.

Known as nacreous clouds, the spectacular formations showing delicate wisps of colors were photographed in the sky over an Australian

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- Evaluation must be done on test data that are independent of the training data, i.e., training and test sets are disjoint.
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- Measures: Precision, recall, F_1, classification accuracy

Precision P and recall R

	in the class	not in the class
predicted to be in the class	true positives (TP)	false positives (FP)
predicted to not be in the class	false negatives (FN)	true negatives (TN)

TP, FP, FN, TN are counts of documents. The sum of these four counts is the total number of documents.

$$\begin{aligned} \text{precision:} \quad P &= TP/(TP+FP) \\ \text{recall:} \quad R &= TP/(TP+FN) \end{aligned}$$

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- In most application scenarios, we need both good precision and good recall.
- So we need to find a good precision-recall tradeoff.

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- The harmonic mean is a kind of "soft" minimum.

Accuracy

$$\text{accuracy} = \frac{\text{TP} + \text{TN}}{\text{TP} + \text{TN} + \text{FP} + \text{FN}}$$

F_1 scores for Naive Bayes vs. other methods

(a)

	NB	Rocchio	kNN		SVM
micro-avg-L (90 classes)	80	85	86		89
macro-avg (90 classes)	47	59	60		60

(b)

	NB	Rocchio	kNN	trees	SVM
earn	96	93	97	98	98
acq	88	65	92	90	94
money-fx	57	47	78	66	75
grain	79	68	82	85	95
crude	80	70	86	85	89
trade	64	65	77	73	76
interest	65	63	74	67	78
ship	85	49	79	74	86
wheat	70	69	77	93	92
corn	65	48	78	92	90
micro-avg (top 10)	82	65	82	88	92
micro-avg-D (118 classes)	75	62	n/a	n/a	87

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Naive Bayes does pretty well, but some methods beat it consistently (e.g., SVM).

Confusion matrix for Reuters-21578

true class: / assigned class:	*money-fx*	*trade*	*interest*	*wheat*	*corn*	*grain*
money-fx	95	0	10	0	0	0
trade	1	1	90	0	1	0
interest	13	0	0	0	0	0
wheat	0	0	1	34	3	7
corn	1	0	2	13	26	5
grain	0	0	2	14	5	10

Example: 14 documents from *grain* were incorrectly assigned to *wheat*.

Exercise

Compute precision, recall and F_1:

	in class	not in class
predicted to be in class	TP: 18	FP: 2
predicted not to be in class	FN: 82	TN: 1,000,000,000

$$\begin{aligned}
\text{precision:} \quad P &= TP/(TP+FP) \\
\text{recall:} \quad R &= TP/(TP+FN) \\
F_1 &= \frac{2PR}{P+R}
\end{aligned}$$

Besonders klausurrelevant

- What is text classification?
  (or: What is sentence classification?)
- Naive Bayes classification rule
- Estimation of Naive Bayes priors and conditionals
- Theory: Bag of words model
  - Maximum likelihood
  - Add-one = Laplace
- Precision, recall, F_1
- Precision-recall tradeoff
- Confusion matrix