

The CYK Parser

The CYK Membership Algorithm

Input:

- Grammar G in Chomsky Normal Form
- String w

Output:

find if $w \in L(G)$

The Algorithm

Input example:

- Grammar G :
 - $S \rightarrow AB$
 - $A \rightarrow BB$
 - $A \rightarrow a$
 - $B \rightarrow AB$
 - $B \rightarrow b$

- String w : $abbba$

abbb

a

a

b

b

b

aa

ab

bb

bb

ab

abb

bbb

abb

abbb

abbb

$$S \rightarrow AB$$

$$A \rightarrow BB$$

$$A \rightarrow a$$

$$B \rightarrow AB$$

$$B \rightarrow b$$

a	a	b	b	b
A	A	B	B	B

aa ab bb bb

aab abb bbb

aabb abbb

aabbbb

$$S \rightarrow AB$$

$$A \rightarrow BB$$

$$A \rightarrow a$$

$$B \rightarrow AB$$

$$B \rightarrow b$$

a	a	b	b	b
A	A	B	B	B

aa	ab	bb	bb
----	----	----	----

S,B	A	A
-----	---	---

aab	abb	bbb
-----	-----	-----

aabb	abbb
------	------

aabbb

$$S \rightarrow AB$$

$$A \rightarrow BB$$

$$A \rightarrow a$$

$$B \rightarrow AB$$

$$B \rightarrow b$$

a	a	b	b	b
A	A	B	B	B

aa	ab	bb	bb
----	----	----	----

S,B	A	A
-----	---	---

aab	abb	bbb
-----	-----	-----

S,B	A	S,B
-----	---	-----

aabb	abbb
------	------

A	S,B
---	-----

aabbb

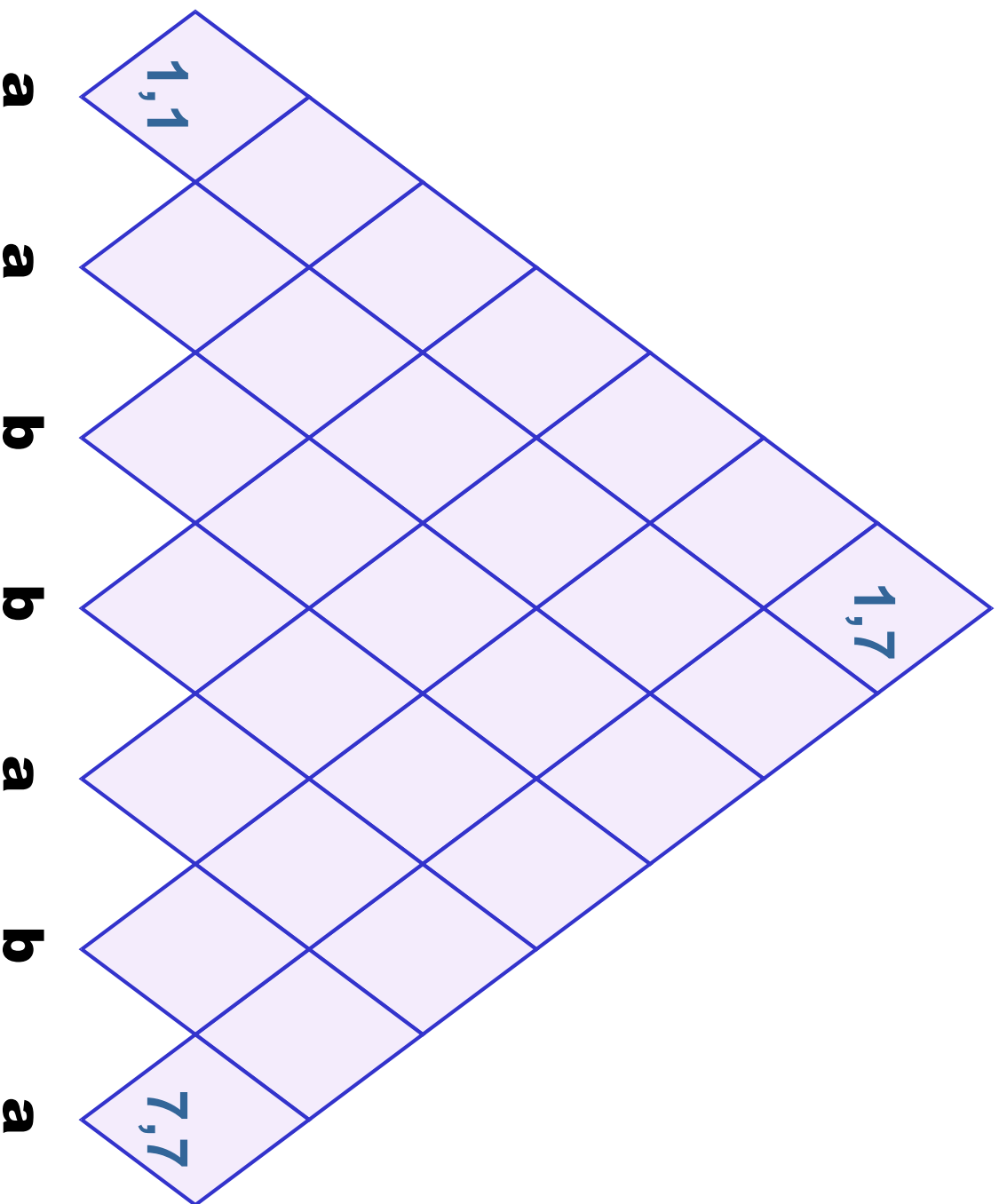
S,B

Therefore: $aabb \in L(G)$

Time Complexity: $|w|^3$

Observation: The CYK algorithm can be easily converted to a parser (bottom up parser)

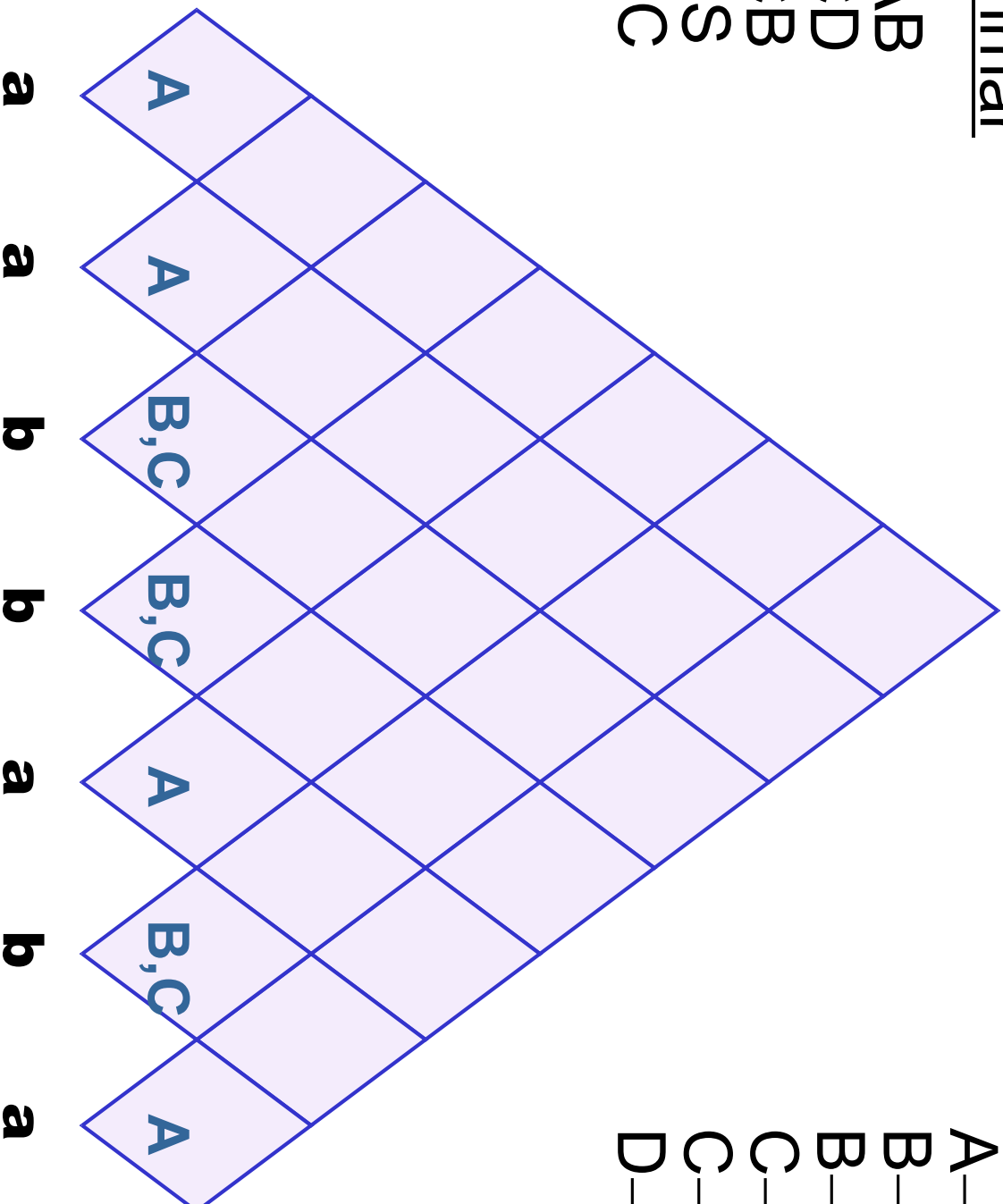
The following slides are courtesy of
Professor Papp, University of Debrecen.



Grammar

$S \rightarrow AB$
 $S \rightarrow CD$
 $S \rightarrow CB$
 $S \rightarrow SS$
 $A \rightarrow BC$

$A \rightarrow a$
 $B \rightarrow SC$
 $B \rightarrow b$
 $C \rightarrow DD$
 $C \rightarrow b$
 $D \rightarrow BA$

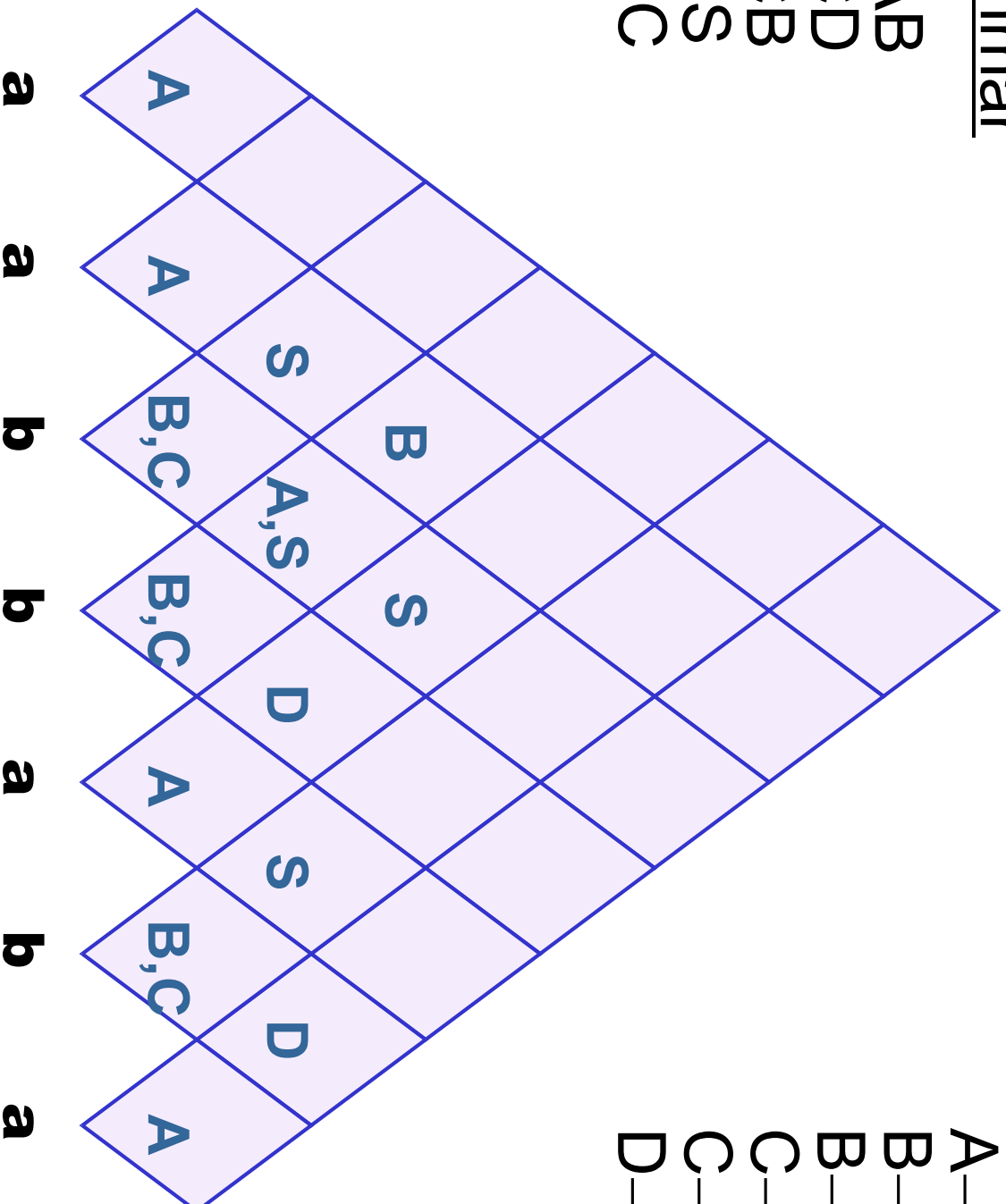


Grammar

Grammar

$S \rightarrow AB$
 $S \rightarrow CD$
 $S \rightarrow CB$
 $S \rightarrow SS$
 $A \rightarrow BC$

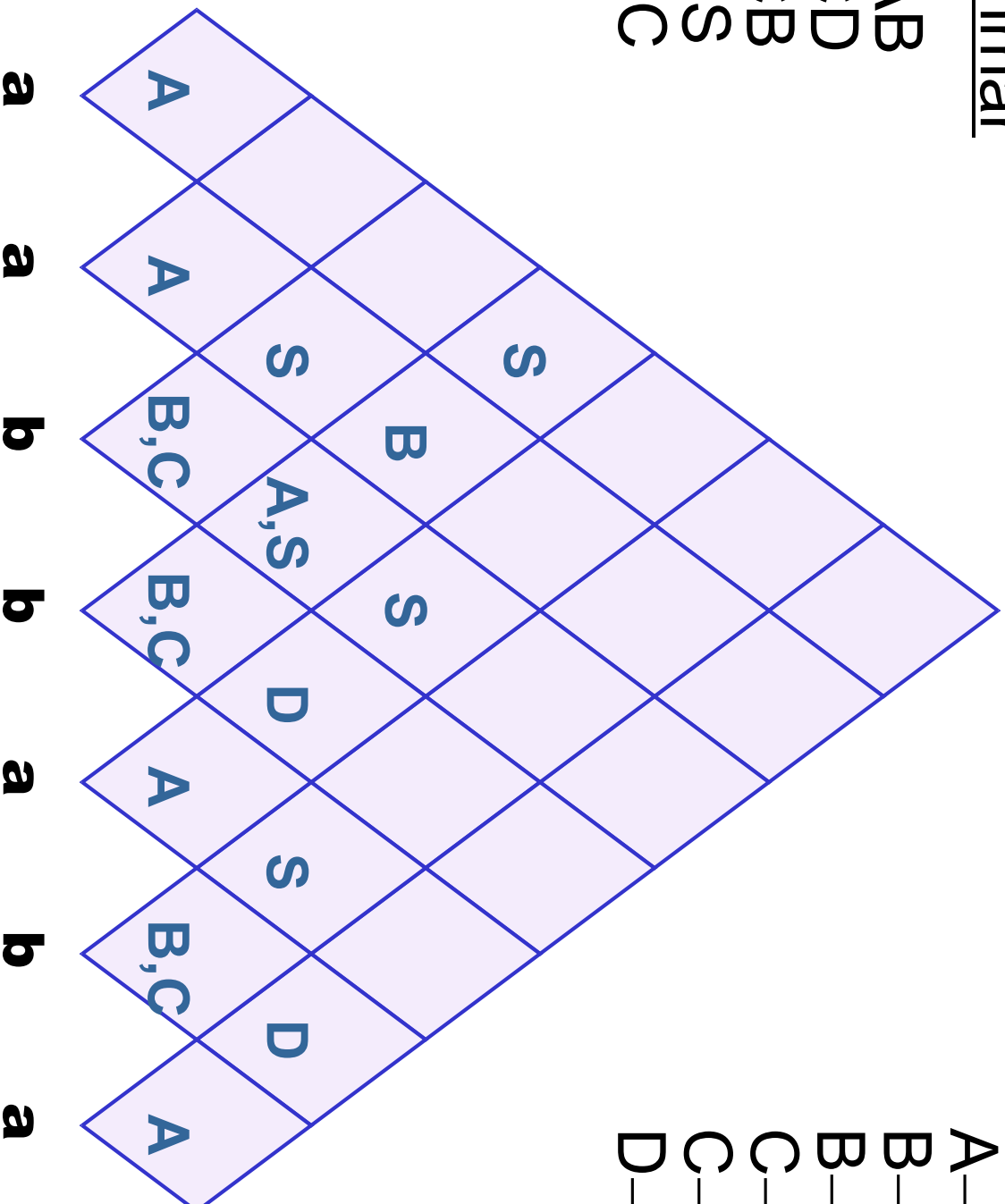
$A \rightarrow a$
 $B \rightarrow SC$
 $B \rightarrow b$
 $C \rightarrow DD$
 $C \rightarrow b$
 $D \rightarrow BA$



Grammar

$S \rightarrow AB$
 $S \rightarrow CD$
 $S \rightarrow CB$
 $S \rightarrow SS$
 $A \rightarrow BC$

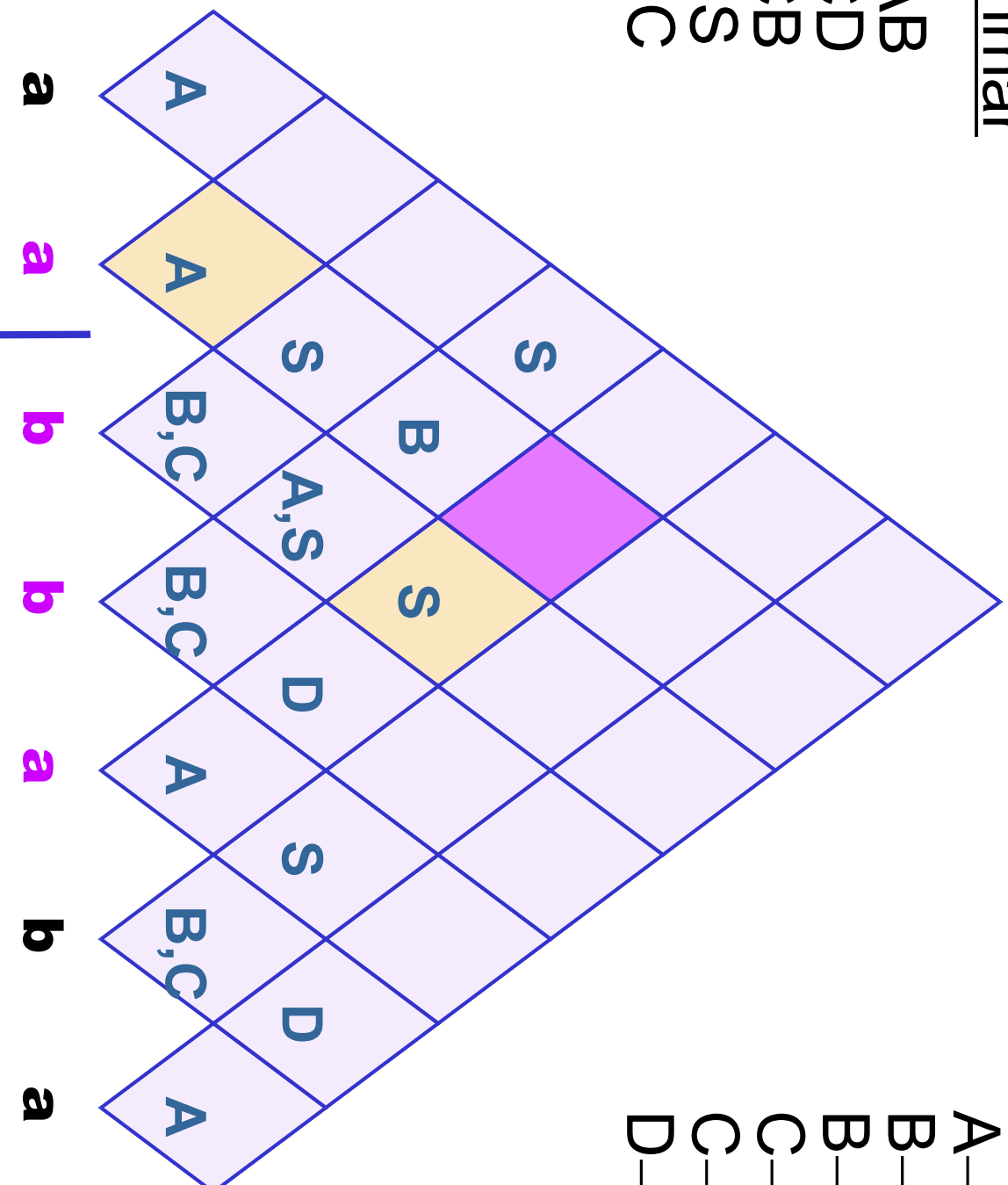
$A \rightarrow a$
 $B \rightarrow SC$
 $B \rightarrow b$
 $C \rightarrow DD$
 $C \rightarrow b$
 $D \rightarrow BA$



Grammar

$S \rightarrow AB$
 $S \rightarrow CD$
 $S \rightarrow CB$
 $S \rightarrow SS$
 $A \rightarrow BC$

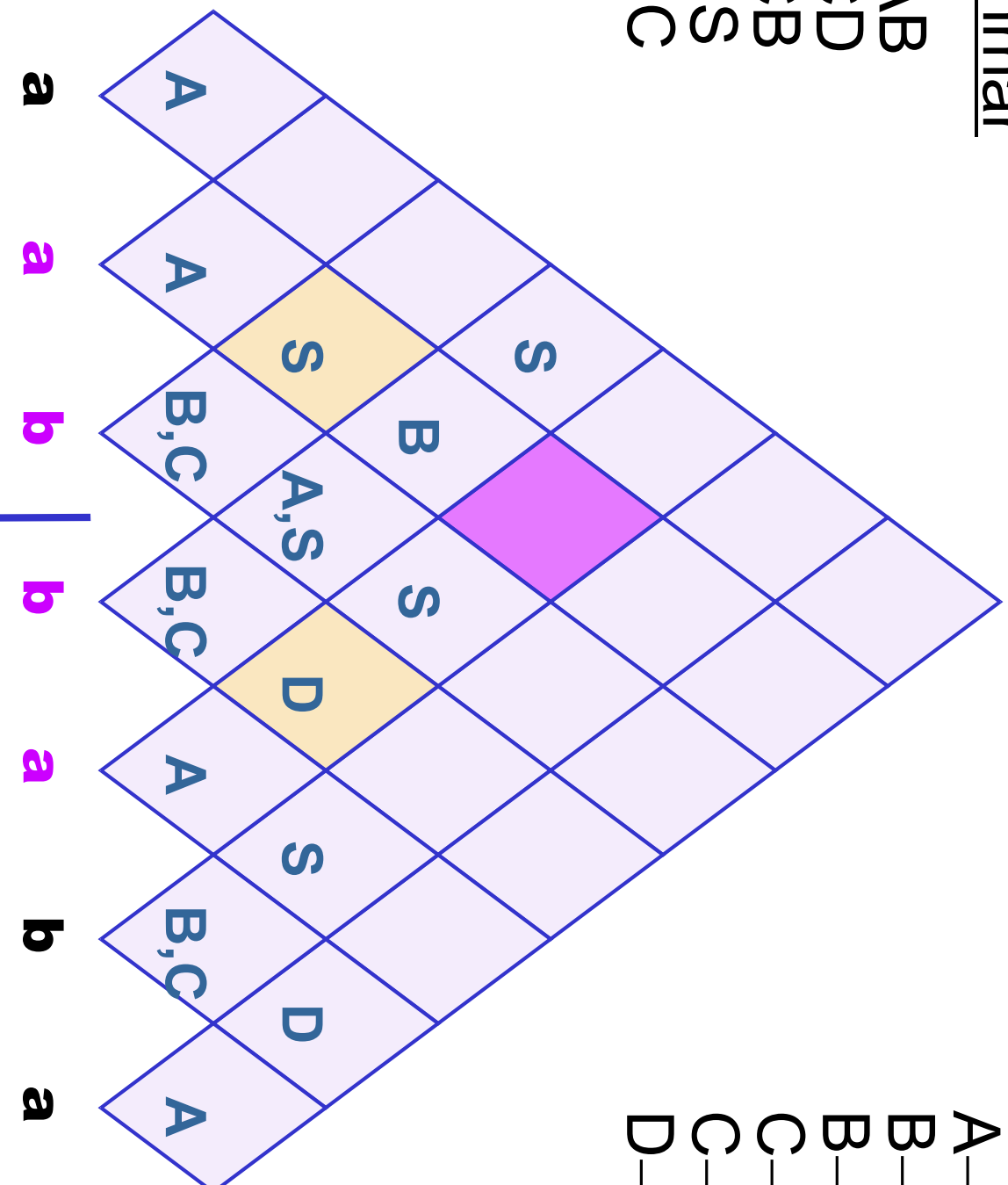
$A \rightarrow a$
 $B \rightarrow SC$
 $B \rightarrow b$
 $C \rightarrow DD$
 $C \rightarrow b$
 $D \rightarrow BA$



Grammar

$S \rightarrow AB$
 $S \rightarrow CD$
 $S \rightarrow CB$
 $S \rightarrow SS$
 $A \rightarrow BC$

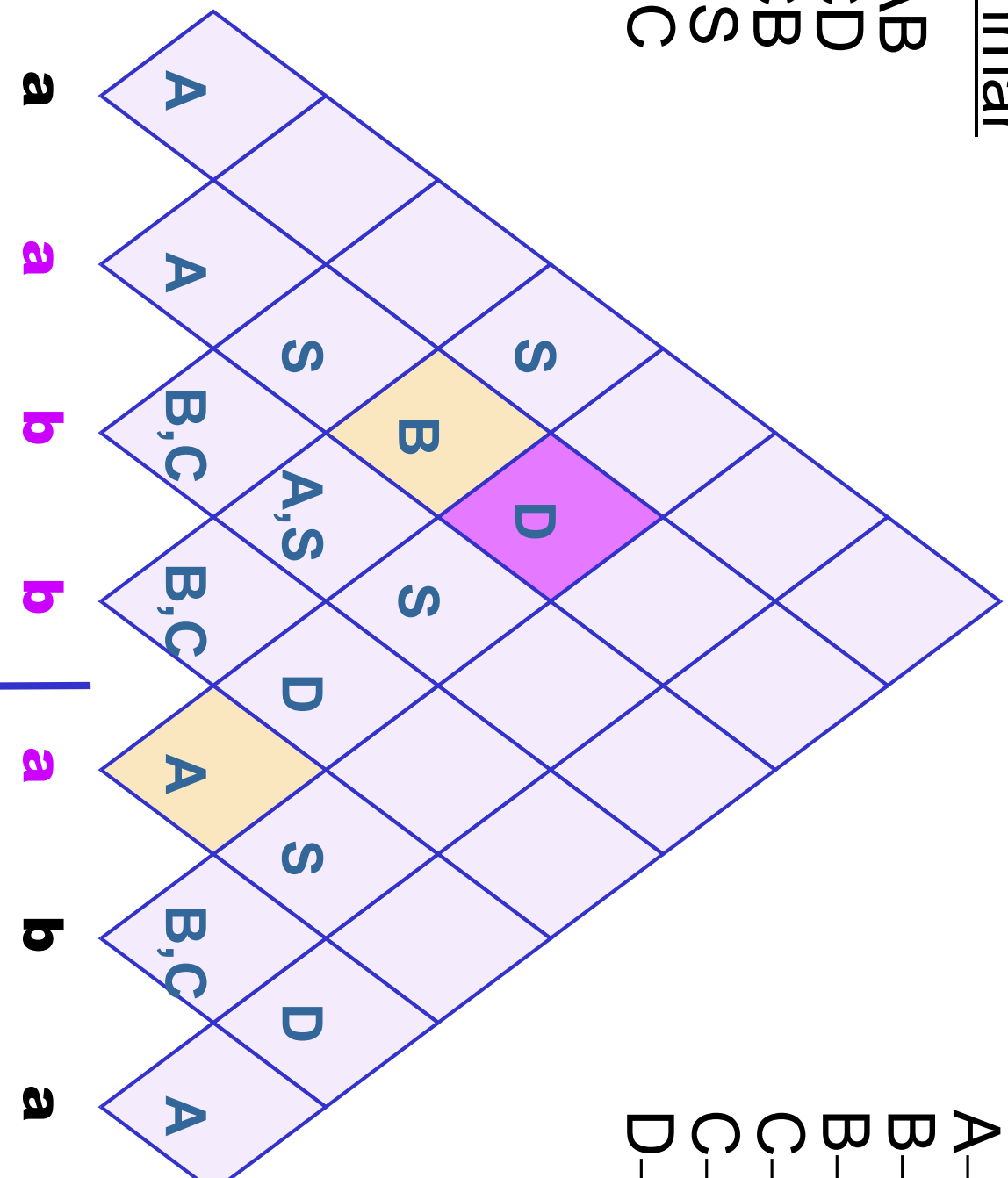
$A \rightarrow a$
 $B \rightarrow SC$
 $B \rightarrow b$
 $C \rightarrow DD$
 $C \rightarrow b$
 $D \rightarrow BA$



Grammar

$S \rightarrow AB$
 $S \rightarrow CD$
 $S \rightarrow CB$
 $S \rightarrow SS$
 $A \rightarrow BC$

$A \rightarrow a$
 $B \rightarrow SC$
 $B \rightarrow b$
 $C \rightarrow DD$
 $C \rightarrow b$
 $D \rightarrow BA$

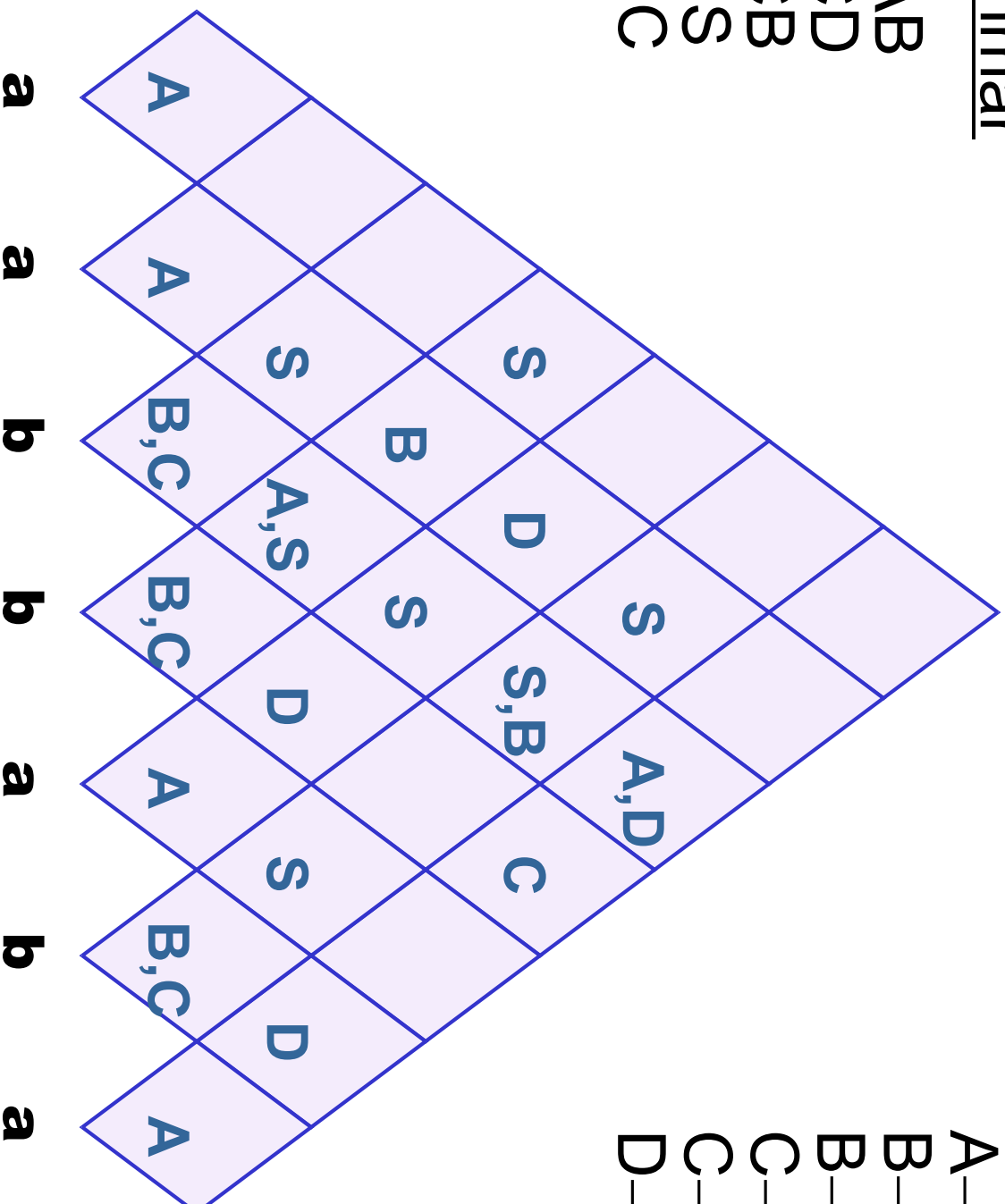


Grammar

Grammar

$S \rightarrow AB$
 $S \rightarrow CD$
 $S \rightarrow CB$
 $S \rightarrow SS$
 $A \rightarrow BC$

$A \rightarrow a$
 $B \rightarrow SC$
 $B \rightarrow b$
 $C \rightarrow DD$
 $C \rightarrow b$
 $D \rightarrow BA$



Grammar

$S \rightarrow AB$

$S \rightarrow CD$

$S \rightarrow CB$

$S \rightarrow SS$

$A \rightarrow BC$

$A \rightarrow a$

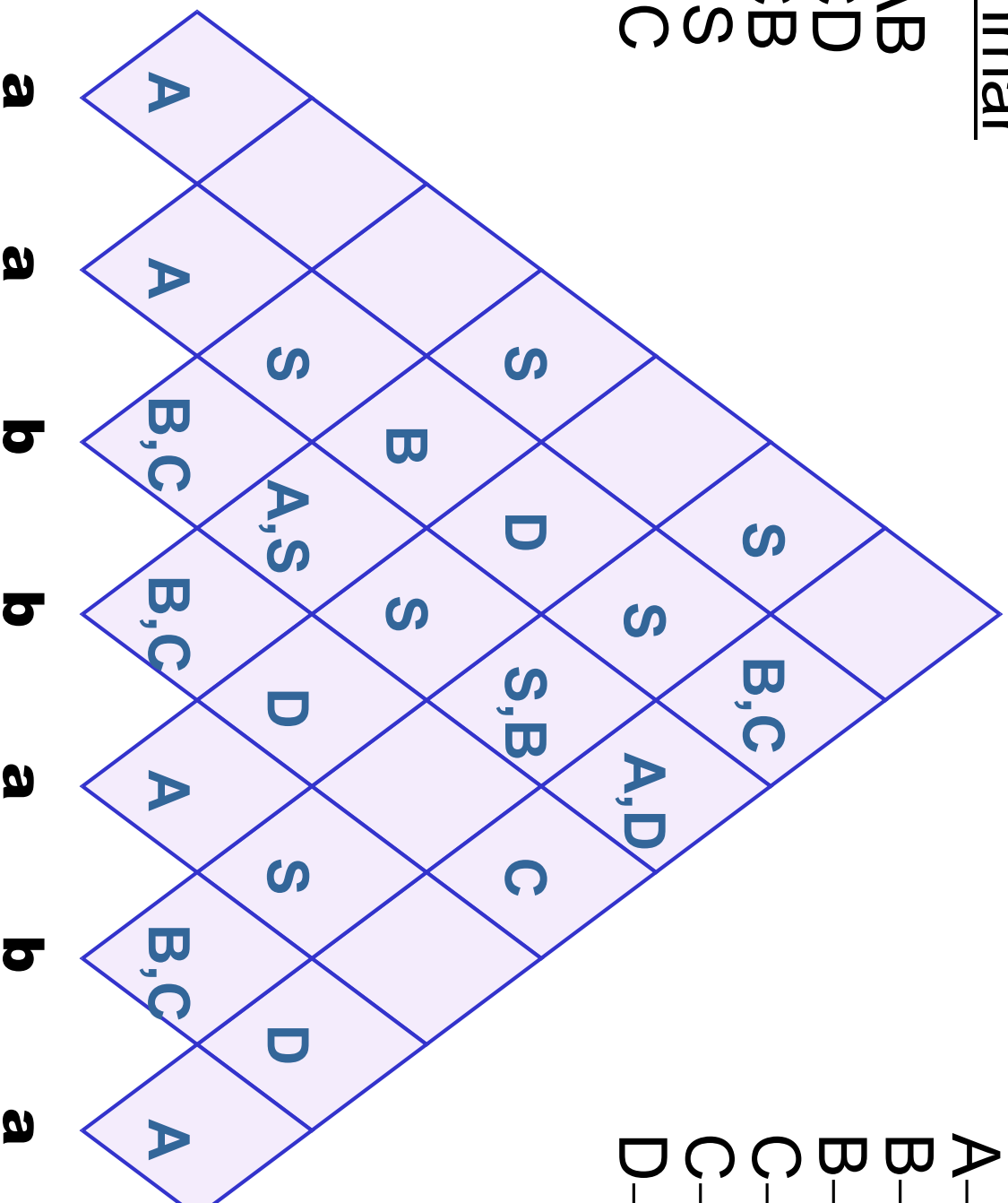
$B \rightarrow SC$

$B \rightarrow b$

$C \rightarrow DD$

$C \rightarrow b$

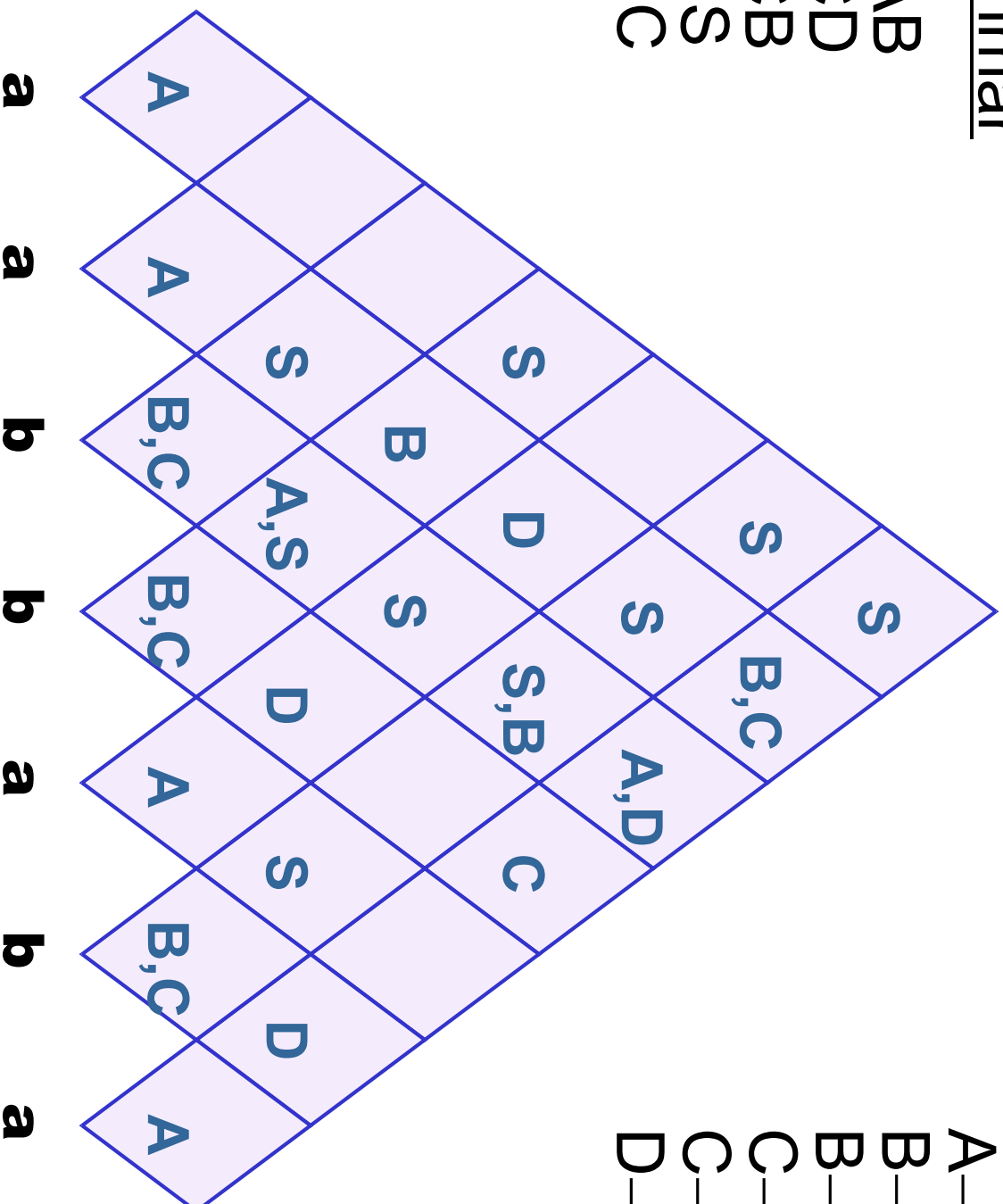
$D \rightarrow BA$



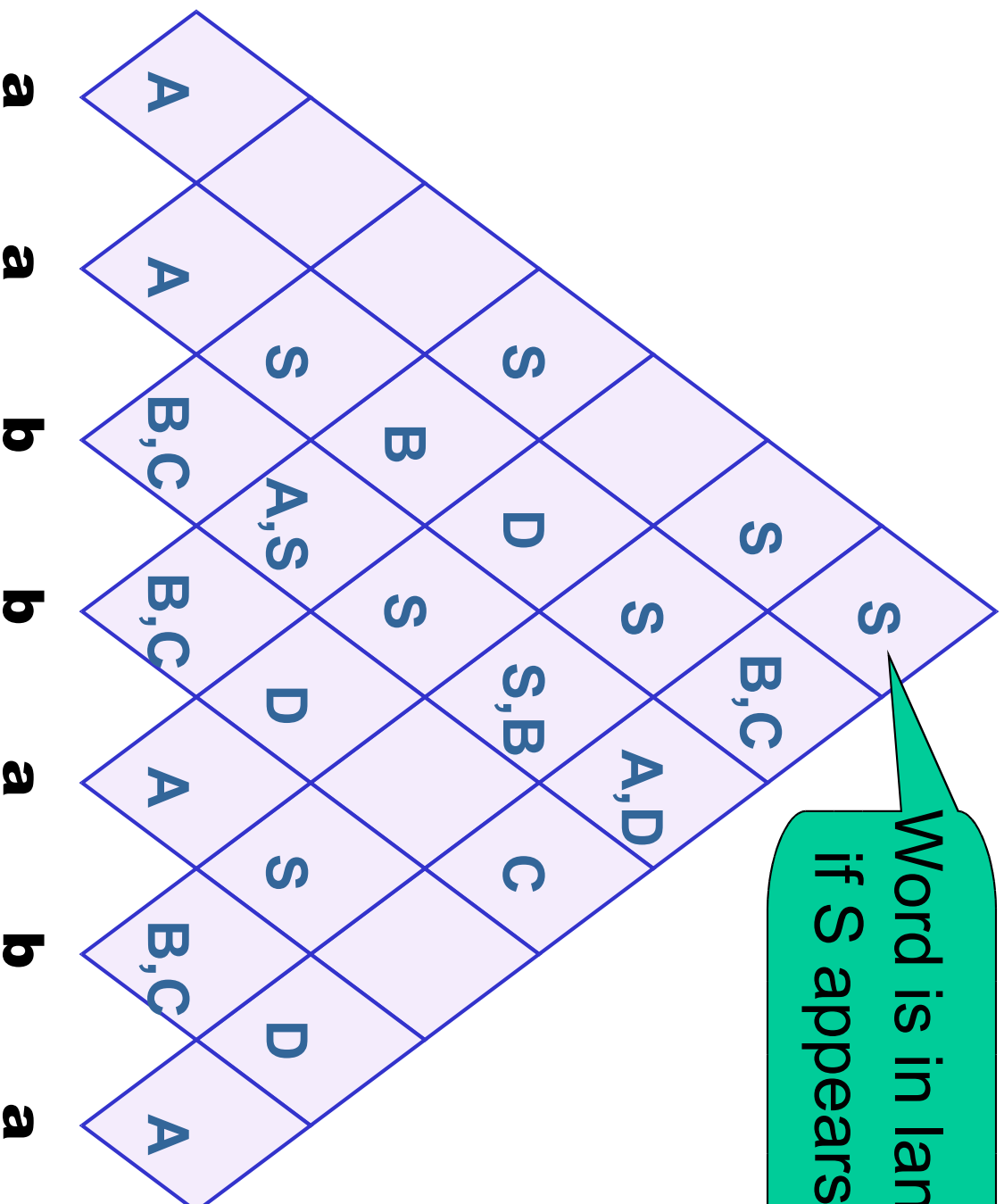
Grammar

$S \rightarrow AB$
 $S \rightarrow CD$
 $S \rightarrow CB$
 $S \rightarrow SS$
 $A \rightarrow BC$

$A \rightarrow a$
 $B \rightarrow SC$
 $B \rightarrow b$
 $C \rightarrow DD$
 $C \rightarrow b$
 $D \rightarrow BA$



Word is in language
if S appears here



Revisit complexity

exercise

Parse “baaba” for this grammar

$$\begin{array}{l} S \rightarrow AB \mid BC \\ A \rightarrow BA \mid a \\ B \rightarrow CC \mid b \\ C \rightarrow AB \mid a \end{array}$$

(Hopcroft, Motwani, Ullman, p301)